A simple stability study for a biped walk with under and over actuated phases

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Abstract

We numerically study the orbital stability with Poincaré map in seven dimensions of a cyclic dynamically stable gait which is composed of single and double support phases and impacts. Physical constraints as ground reactions and limited torques are taken into account with the control. The double support is used to improve the stability of the biped. Numerical tests are presented, where the maximum modulus of the eigenvalues of the linearized Poincaré map around the fixed point of the periodic motion is checked with the power method to be less than one, to ensure stability.

1 Introduction

To understand better the effects of the gravity on the dynamically stable gait the study of walking bipeds without feet is an active research area currently, see [1, 2, 3, 4, 6, 13]. The main difficulty is that in single support phase, the biped is under actuated. During this phase there is no equilibrium point which is asymptotically stable. It is necessary to introduce the concept of orbital asymptotical stability, mainly studied with Poincaré method. Poincaré method consists in studying the stability of Poincaré map, the application which give the intersection of the orbit with a surface for a given previous intersection point. In [4] a linearization of the Poincaré map is calculated analytically. In [5] the linearization is calculated numericlally. In [7] this concept of orbital stability and Poincaré method was extended to system with impulse effects, viewed that usually the impact of the swing leg with the ground is considered as impulsive. In [7] the dimension of Poincaré map is reduced to one, which greatly simplify the study.
Some authors increased the convergence to the periodic orbit by modifying the length step or the inclination of the trunk from one step to the next step, see [1]. Other authors used a double support phase, which also allows to increase stability. In the paper [14] a walking gait with single and double support phases is designed for a biped without feet. This reference gait is tested with the robot Sony AIBO. With this strategy using a double support phase for which the biped is over actuated they are able to start from a stop phase.

We propose also to include a double support phase here to design a cyclic gait easier and to increase the convergence velocity to a cyclic stable gait without any decoupled system. In [11] we showed that for a problem where the Poincaré map is reduced to a one dimension, there is a zone of one step convergence. To design this zone the constraints of limited torques, no take off, no slipping are taken into account. In the present paper we extent the stability study to the whole state of the biped, but we only study local stability around reference motion. We consider the numerical linearization of a Poincaré map of dimension seven. We use the physical data of RABBIT, a five link experimental biped without feet, which is presented in [3]. We designed bipedal cyclic gaits which are composed of a single support and a double support phases. We defined parameters for this gait which are chosen with an optimization process, see [10]. The unilateral constraints are taken into account in the definition of the motion. We designed a PD based controller satisfying the unilateral and torque constraints.

The organization of the paper is the following. Section 2 details the biped presentation. Afterwards the dynamic model in single and double support phases and the impact equations are recalled in this section. The definition of the motion is done Section 3. All the classes of constraints (geometric, ground reactions, limited torque value of the actuators, constraint introduced by the definition of the motion) are described in this Section. The control strategy for both phases are presented Section 4. The Poincaré method to study stability is recalled in Section 5. We detail the calculus of the maximum modulus of the eigenvalues of the linearized Poincaré map around the fixed point of the periodic motion. All numerical tests are discussed Section 6. Finally we offer our conclusion and perspectives Section 7.

2 Biped presentation and model

Figure 1 presents a diagram of the studied biped. This biped is only moving in the sagittal plan, composed of 5 links (2 tibias, 2 femurs and the trunk), 4 actuated joints (the 2 knees and the hips), and does not have feet. We note $\Gamma = [\Gamma_1\,\Gamma_2\,\Gamma_3\,\Gamma_4]^T$ the torque vector, $\delta = [\delta_1\,\delta_2\,\delta_3\,\delta_4]^T$ the actuated joint variables, $q = [\alpha\,\delta^T]^T$ the joint variables with orientation of the biped in space, and $X = [q^T, x_t, y_t]^T$ the configuration, orientation and position vector, where (x_t, y_t) is the position of the center of gravity of the trunk. $R_i = [R_{ix}\,R_{iy}]^T$, $(i = 1, 2)$ are the ground reaction forces respectively on feet 1 and 2.

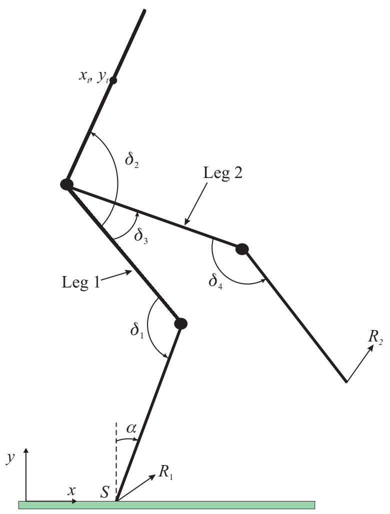

Figure 1: *Five link Biped's diagram: Generalized Coordinates, Torques and Forces Applied to the Leg Tips.*

The dynamic model (1) of the biped is obtained from Lagrange's equations.

$$A(q)\ddot{X} + H(q, \dot{q}) = D_\Gamma \Gamma + D_1(q)R_1 + D_2(q)R_2 \quad (1)$$

$A(7 \times 7)$ is the symmetric positive definite inertia matrix, $H(7 \times 1)$ represents the Coriolis, centrifugal and gravity effects, $D_\Gamma(7 \times 4)$ is a simple matrix composed of 1 and 0, $D_i(7 \times 2)$, $(i = 1, 2)$ are jacobian matrices linking extremities of feet and joints.
The biped is also subject to position constraints (2), depending on whether feet are on the ground or not. These constraints are also written with respect to velocities and acceleration, by derivation of position constraints.

$$\begin{cases} d_i(X) = constant, \quad V_i = D_i(q)^T \dot{X} = 0 \\ \dot{V}_i = D_i(q)^T \ddot{X} + H_i(q, \dot{q}) = 0 \qquad i = 1, 2 \end{cases} \quad (2)$$

$d_i(X)(2 \times 1)$ and $V_i(2 \times 1)$ are respectively the position and velocity of the foot i.
The models for the single support phase, the double support phase and the impact are restrictions obtained from the general model presented here.
During the single support phase, one feet of the biped is on the ground. Let us consider the case of the foot 1 on the ground and the foot 2 in the air. The model of single support is then represented by dynamic model (1) with no effort on foot 2, i.e. $R_2 = [0\ 0]^T$, and constraint equations (2) for foot 1, i.e. $i = 1$. The case for the foot 2 on the ground is similar. Let us note that the biped is under actuated during this phase since there are four actuators and five degrees of freedom.
During the double support phase, both feet are on the ground. Then the model for the double support phase is given by dynamic model (1) with constraint equations (2) for both feet, i.e. $(i = 1, 2)$. Let us note that the biped is over actuated during this phase since there are three degrees of freedom and four actuators.
We design our nominal reference gait without impact. However we need to take into account the impacts because they occur due to perturbations. The impact is considered rigid, passive, instantaneous, with impulsive ground reactions, and with a null restitution coefficient. The model of impact (3) is then obtained from integration of dynamic model (1) between instants just before impact and just after impact, see [8].

$$A(q)(\dot{X}^+ - \dot{X}^-) = D_1(q)I_{R_1} + D_2(q)I_{R_2} \quad (3)$$

I_{R_1} and I_{R_2} are the impulsive ground reactions for both feet. The notation $^-$ means just before impact and $^+$ means just after impact. Additional equations are given by the contact laws. We suppose that the two feet remain on the ground without moving after impact.

$$D_i(q)^T \dot{X}^+ = 0 \qquad i = 1, 2 \quad (4)$$

Then the calculation of velocities after impact and impulsive ground reactions are given by (5).

$$\begin{bmatrix} \dot{X}^+ \\ I_{R_1} \\ I_{R_2} \end{bmatrix} = A_{impact}^{-1} \begin{bmatrix} A\dot{X}^- \\ 0 \\ 0 \end{bmatrix} \quad (5)$$

where,

$$A_{impact} = \begin{bmatrix} A & -D_1 & -D_2 \\ D_1^T & 0 & 0 \\ D_2^T & 0 & 0 \end{bmatrix} \quad (6)$$

In simulation we consider that impact occurs when altitude of swing foot become zero.

3 Reference Motion Definition

We consider one periodic step composed of a double support phase, a single support phase and an impact. There is an infinite number of parameters to define reference motions. We have chosen to restrict the motion definition to a finite number of parameters by taking polynomials for the reference motion.
In single support, the biped is under actuated and we will then define the trajectory of as much variables as actuated joints. For control purposes it is easier to define trajectories for the four actuated joints angles. Due to the under actuation, we will also take these joint variables as functions of ankle angle α. In spite of under actuation, this allows to define all the configurations during the walk. The speed at which all these configurations are tracked in single support will follow from the dynamic behavior of the biped. The form of the four actuated joints as polynomials of α is the following $(i = 1, ..., 4)$:

$$\delta_{i,ss}(\alpha) = a_{i0} + a_{i1}\alpha + a_{i2}\alpha^2 + a_{i3}\alpha^3 + a_{i4}\alpha^4. \quad (7)$$

In double support, due to the over actuation, we define the motion of as much variables as degrees of freedom. We firstly define α in order to directly tune during double support the dynamics of this angle, as its dynamics are not controlled in single support. α is defined as a polynomial of time:

$$\alpha_{ds}(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3. \quad (8)$$

For an homogenous definition with single support, we define the motion of two other actuated joints as polynomials of α $(i = 1, 2)$:

$$\delta_{i,ds}(\alpha) = a_{i0} + a_{i1}\alpha + a_{i2}\alpha^2 + a_{i3}\alpha^3. \quad (9)$$

The coefficients of the polynomials for a reference motion will be obtained by an optimization process. First we consider the parameters that are the limit conditions for each polynomial. We consider position and velocity at the beginning and at the end of each phase and an additional position during single support phase. Then we reduce these parameters of the reference motion to a minimum set of parameters. This is due to the fact that there are continuity conditions between phases that link parameters of each phases. For our walking motion we obtained 17 parameters for the optimization process, that are the initial and final joint angles of double support for the controlled joints, the final velocities of double support for the controlled joints, the final velocities of single support for the controlled joints, the intermediate configuration of single support, and the step length.
The optimization criteria (10) is:

$$C = \frac{1}{d} \int_0^{T_{step}} \Gamma^T \Gamma dt \quad (10)$$

where d is the step size and T_{step} the step duration.
During the walk, some constraints must be satisfied. We can distinguish physical constraints and constraints on the motion of the biped. Physical constraints (11) include no take off and no sliding of the feet on the ground as well as torque limits.

$$\begin{cases} R_{iy} > 0, \quad |R_{ix}/R_{iy}| < f \quad i = 1, 2 \\ -\Gamma_{max} \leq \Gamma_i \leq \Gamma_{max} \quad i = 1, ..., 4 \end{cases} \quad (11)$$

f is the friction coefficient between the feet and the ground and Γ_{max} is the torque limitation.
The constraints we considered on the motion are that the trajectory of the free feet must be above a parabola and that the trunk must be erected.
The optimization problem composed of the criteria (10) and of the physical constraints (11) and the constraints on the motion is non linear. It is solved by a sequential quadratic programming method.

4 Control of the biped

The difficult aspects of the control of this biped are to satisfy the physical constraints during the walk, even if the reference motion satisfies them. Indeed, especially during the double support phase, these constraints stay close to the reference motion. Another difficult aspect is to deal with the under actuation of the biped during the single support phase. For this reason, in part 3 we choose to express the reference motions in function of the non actuated angle α instead of time, in order to define all successive configurations of the biped. For the control we will also use α instead of time so that the convergence rate of the controlled joints will be the same over one step, whatever the dynamics of α. This can be done since the evolution of α is monotonic.
In the following parts, we will linearize the controlled system by inverting the dynamics, write the physical constraints in terms of the inputs and finally we will present a PD based control law that takes into account these constraints.

4.1 Input-Output Linearization of the System

For the actual biped model (1) the inputs are the torques and the outputs are the positions and veloc-

ities. We will write the needed torques in terms of the desired acceleration of the joint variables.
In single support the model (1) is written in terms of the second derivatives in time of the joint variables. We can firstly reduce the total acceleration vector $\ddot{X}$ with the contact constraint (2) in acceleration for the foot on the ground.

$$\ddot{X} = A_{x,ss}(q)\ddot{q} + B_{x,ss}(q) \tag{12}$$

$A_{x,ss}$ is a (7×5) matrix and $B_{x,ss}$ is a (7×1) vector. We control the four actuated joints with respect to α, which is not controlled. Then we rewrite these joint variable velocities and accelerations in terms of the first and second derivatives in α of the joint variables.

$$\left\{ \dot{\delta}_i = \frac{\partial \delta_i}{\partial \alpha}\dot{\alpha},\ \ddot{\delta}_i = \frac{\partial^2 \delta_i}{\partial \alpha^2}\dot{\alpha}^2 + \frac{\dot{\delta}_i}{\dot{\alpha}}\ddot{\alpha} \quad i = 1, ..., 4 \right. \tag{13}$$

Finally we invert the dynamic model (1) considering the relations (12) and (13). We obtain the following expression that gives the torque Γ needed to obtain some desired second derivative in α of the actuated joint variables, $\frac{\partial^2 \delta}{\partial \alpha^2}$.

$$\begin{bmatrix} \ddot{\alpha} \\ \Gamma \\ R_1 \end{bmatrix} = A_{int,ss}(q, \dot{q})\frac{\partial^2 \delta}{\partial \alpha^2} + B_{int,ss}(q, \dot{q}) \tag{14}$$

$A_{int,ss}$ is a (7×4) matrix and $B_{int,ss}$ is a (7×1) vector. This inversion also allows to calculate the ground reactions R_1 and the acceleration $\ddot{\alpha}$ that will be obtained.
By feeding the torque computed with (14) in the dynamic model (1) we obtain a double integrator system with inputs $\frac{\partial^2 \delta}{\partial \alpha^2}$ and outputs δ, $\frac{\partial \delta}{\partial \alpha}$, and some nonlinear dynamics in α depending on $\frac{\partial^2 \delta}{\partial \alpha^2}$, $\frac{\partial \delta}{\partial \alpha}$ and δ.
In double support we follow the same steps as for the linearization of the model in single support. The biped has three degrees of freedom, due to the contact constraints (2) for both feet. We firstly express the total vector $\ddot{X}$ of acceleration with respect to the independent accelerations, with the contact constraints (2) for both feet.

$$\ddot{X} = A_{x,ds}(q)\begin{bmatrix} \ddot{\alpha} \\ \ddot{\delta}_1 \\ \ddot{\delta}_2 \end{bmatrix} + B_{x,ds}(q) \tag{15}$$

$A_{x,ds}$ is a (7×3) matrix and $B_{x,ds}$ is a (7×1) vector. We then express second derivatives in time with respect to first and second derivatives in α. For the control of α we will control the dynamics of α, i.e. $\dot{\alpha}$ and then $\ddot{\alpha}$ is written in function of first derivative in α of $\dot{\alpha}$.

$$\ddot{\alpha} = \frac{d\dot{\alpha}}{d\alpha}\dot{\alpha} \tag{16}$$

For δ_1 and δ_2 these relations are the same as for the ones in single support phase (13) with $i = 1, 2$.
We now invert the dynamic model (1) considering the double support relations (15), (16) and (13).

$$\begin{bmatrix} \Gamma \\ R_{2y} \\ R_1 \end{bmatrix} = A_{int,ds}(q, \dot{q})\begin{bmatrix} \frac{d\dot{\alpha}}{d\alpha} \\ \frac{\partial^2 \delta_1}{\partial \alpha^2} \\ \frac{\partial^2 \delta_2}{\partial \alpha^2} \\ R_{2x} \end{bmatrix} + B_{int,ds}(q, \dot{q}) \tag{17}$$

It appears that this inversion depends on another parameter than the desired derivatives in α, due to the fact that the biped is over actuated. We choose R_{2x} the normal ground reaction of the foot 2 as this additional parameter. R_{2x} can be considered as a fourth input of the linearized system. It will be determined by the constraint control presented section 4.2. But if R_{2x} is not unique the one minimizing the torque norm will be chosen.
By feeding the calculated torque Γ with (17) into the biped represented by dynamic model (1) we obtain a double integrator linear system with inputs $\frac{\partial \dot{\alpha}}{\partial \alpha}$ and $\frac{\partial^2 \delta_{1,2}}{\partial \alpha^2}$ and with outputs δ_1, δ_2, $\dot{\alpha}$, $\frac{\partial \delta_1}{\partial \alpha}$, $\frac{\partial \delta_2}{\partial \alpha}$.
For the inversion of models in single support and double support, we never observed any singularity, which can be due to the fact that the reference motions obtained by optimization are far from singularities.
In simulation the commutation from the control of single support to the control of double support occurs when the high of the swing foot become zero, but in experiment it would be better to detect the ground force peak of impact. The other control commutation occurs when the desired final ankle angle of double support is reached.

4.2 Constraints

Physical constraints can be rewritten in function of the control inputs. Here are the original constraints (11) rewritten with some margins.

$$\left\{ \begin{array}{l} R_{iy} \geq R_{iy,min},\ -R_{iy} \leq \frac{R_{ix}}{f_{max}} \leq R_{iy}, \\ -\Gamma_{max} \leq \Gamma_j \leq \Gamma_{max},\ i = 1, 2,\ j = 1...4 \end{array} \right. \tag{18}$$

For the robustness margins, we have $f_{max} < f$, $R_{iy,min} > 0$.

The inverse dynamic equation (14) gives a relation between ground reactions, torque and control inputs in single support. The constraints of single support (inequalities (18) with ground constraints for only one foot) are then rewritten linearly in terms of control inputs.

$$A_{c,ss}(q,\dot{q})\frac{\partial^2\delta}{\partial\alpha^2} \leq B_{c,ss}(q,\dot{q}) \tag{19}$$

$A_{c,ss}$ is a (11×4) matrix and $B_{c,ss}$ is a (11×1) vector. Similarly, constraints of double support (18) are obtained on control inputs using equation (17).

$$A_{c,ds}(q,\dot{q})\begin{bmatrix}\dfrac{d\dot{\alpha}}{d\alpha}\\[2ex] \dfrac{\partial^2\delta_1}{\partial\alpha^2}\\[2ex] \dfrac{\partial^2\delta_2}{\partial\alpha^2}\\[2ex] R_{2x}\end{bmatrix} \leq B_{c,ds}(q,\dot{q}) \tag{20}$$

$A_{c,ds}$ is a (14×4) matrix and $B_{c,ds}$ is a (14×1) vector.

4.3 Control with Constraints

The principle of this control with constraints is to calculate control inputs with a usual controller and then to modify the less possible the control inputs in order to satisfy the constraints, see [12].
At present, we will note r for the reference motion, d for the desired motion, c for variables that allow to satisfy constraints, and no indices for variables measured.
In single support phase a classical proportional and derivative controller is used.

$$\frac{\partial^2\delta_{i,d}}{\partial\alpha^2} = \frac{\partial^2\delta_{i,r}}{\partial\alpha^2} + k_v\left(\frac{\partial\delta_{i,r}}{\partial\alpha} - \frac{\partial\delta_i}{\partial\alpha}\right) + k_p(\delta_{i,r}-\delta_i) \quad i=1,...,4 \tag{21}$$

We use the norm 2 for the problem of finding the closest control input under constraints. We obtain the following minimization problem.

$$\begin{gathered}\min_{\frac{\partial^2\delta_{dc}}{\partial\alpha^2}\in I\!R^4}\left\|\frac{\partial^2\delta_{dc}}{\partial\alpha^2} - \frac{\partial^2\delta_d}{\partial\alpha^2}\right\|^2\\ A_{c,ss}\frac{\partial^2\delta_{dc}}{\partial\alpha^2} \leq B_{c,ss}\end{gathered} \tag{22}$$

It is a quadratic optimization problem under linear constraints. The solution is then unique.
In double support phase we use a proportional control law for α since the corresponding linear system obtained by linearization is of degree one, and a proportional and derivative law for δ_1 and δ_2.

$$\begin{cases}\dfrac{d\dot{\alpha}_d}{d\alpha} = \dfrac{d\dot{\alpha}_r}{d\alpha} + k_{p\alpha}(\dot{\alpha}_r - \dot{\alpha})\\[2ex] \dfrac{\partial^2\delta_{i,d}}{\partial\alpha^2} = \dfrac{\partial^2\delta_{i,r}}{\partial\alpha^2} + k_v\left(\dfrac{\partial\delta_{i,r}}{\partial\alpha} - \dfrac{\partial\delta_i}{\partial\alpha}\right) + k_p(\delta_{i,r}-\delta_i)\\ \quad i=1,2\end{cases} \tag{23}$$

In double support the problem of control under linear constraints is similar to the one in single support.

$$\begin{gathered}\min_{\left[a_{dc}^T;R_{2x}\right]^T\in I\!R^4}\|a_{dc}-a_d\|^2\\ A_{c,ds}\begin{bmatrix}a_{dc}\\ R_{2x}\end{bmatrix} \leq B_{c,ds}\end{gathered} \tag{24}$$

with $a = \left[\dfrac{d\dot{\alpha}}{d\alpha}, \dfrac{\partial^2\delta_1}{\partial\alpha^2}, \dfrac{\partial^2\delta_2}{\partial\alpha^2}\right]^T$.

The coefficient of the PD and P control laws are tuned in order to obtain a critical aperiodic behavior. The controller is chosen in function of the bandwidth of the mechanical portion of the joints which is approximately 12-Hz for RABBIT.

5 Poincaré Stability Study

The previous control strategy with constraints gives rise to the question of stability with the constraints. In fact, since the reference motion satisfies the constraints, there exist a small space around the reference motion for which constraints are satisfied strictly. So with the presented controller we have asymptotic convergence locally around the reference motion for the controlled degrees. But with the impact phenomenon and the under actuation in single support stability is not assured anymore. Poincaré method is here a way to verify stability locally around reference motion. The Poincaré method for periodic orbits consists in studying the stability of the intersection of the motion with a surface. We call Poincaré map the application that for a given position of the intersection point in the surface gives the next intersection point in the surface. A fixed point of this function corresponds to a periodic motion and it is stable if the maximum modulus of the eigenvalues of the linearized Poincaré map around the fixed point is strictly less than one. In [7] these properties have been generalized for systems with impulse effects, like bipeds.
The stability of the system in closed loop will be simply checked numerically, calculating the maximum modulus of eigenvalues of linearized Poincaré map. To do

so we use the power method (see [9]) that allows to calculate the eigenvalue of largest modulus and the associated eigenvector (called dominant eigenpair). For our problem the power method is slightly different since it combines together the linearization procedure with the dominant eigenpair determination. This method is numerically better than first calculate the linearization of the Poincaré map and then calculate the eigenvalues. Let us note $f : I\!R^n \rightarrow I\!R^n$ the Poincaré map, x_p a fixed point of this function, and the function $g(\Delta x) = f(x_p + \Delta x) - x_p$. The following recursive program with initial value $\Delta x_0 = \Delta x_{max}$, will converge to the dominant eigenpair of the linearization of f around the fixed point x_p.

$$\left\{ \begin{aligned} \Delta y_k &= g(\Delta x_k) \\ \Delta x_{k+1} &= \frac{\Delta y_k}{c_{k+1}} \\ c_{k+1} &= \|\Delta y_k\|_{\Delta x_{max}} \end{aligned} \right. \tag{25}$$

where norm $\|.\|_{\Delta x_{max}}$ is defined by $\|\Delta y_k\|_{\Delta x_{max}} = \frac{\Delta y_{k,j}}{\Delta x_{max,j}}$ where j is the j^{th} component of Δy_k satisfying $\frac{|\Delta y_{k,j}|}{\Delta x_{max,j}} = \max_{1 \leq i \leq n} \left\{ \frac{|\Delta y_{k,i}|}{\Delta x_{max,i}} \right\}$ (i is the i^{th} component of vector Δy_k). Δx_{max} is the vector of maximal values that Δx_k can take so that Poincaré map f can still be linearly approximated with a good precision around the fixed point. For this reason Δx_{max} must be sufficiently small but must not either be too small to obtain a sufficient precision in the calculus of the dominant eigenpair. In practice we will choose first Δx_{max} to be in the linear domain of f and then we will choose the precision of simulation to obtain a good precision of calculus of eigenpair.
c_k will converge to the dominant eigenvalue λ_{max} and Δx_k will converge to the dominant eigenvector $\Delta x_{\lambda_{max}}$. The rate of convergence becomes better with the decrease of the ratio of the second dominant eigenvalue by the dominant eigenvalue.

6 Simulation results

The biped with the control law is simulated with Matlab Simulink.
The surface we consider for the Poincaré study is the instant of beginning of double support, just after the impact. The state of the biped on this surface is described by four configuration parameters α_{ids}, $\delta_{1,ids}$, $\delta_{2,ids}$, d the distance between feet and three velocity parameters $\dot{\alpha}_{ids}$, $\dot{\delta}_{1,ids}$, $\dot{\delta}_{2,ids}$. We note *ids* for initial of double support. The Poincaré map is then a function from $I\!R^7$ to $I\!R^7$. The figures 2 4 and 3 present some results about the dominant eigenvalue calculation.

We can see on figure 2 that convergence takes about six iterations, which is very few. The obtained dominant eigenvalue is $\lambda_{max} = 1\,10^{-3}$, which is very small.

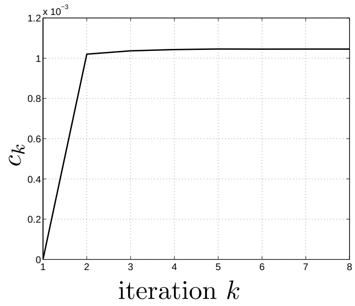

Figure 2: *Convergence of the absolute value of dominant eigenvalue with power method.*

The figures 3 and 4 allow to see if the vector Δx_{max} has been well chosen. The normalized dominant vector corresponds to the abscissa of 1 and we can see that it is in the linear part since the eigenvalues are constant between 0 and 1 except when it tends to 0 where the eigenvalues tend to infinity. This is due to numerical noise that increase when the norm of the eigenvector tends to 0. We choose a precision of simulation of $1\,10^{-9}$, and we can see then that there is not so much numerical noise for calculus of λ_{max}.

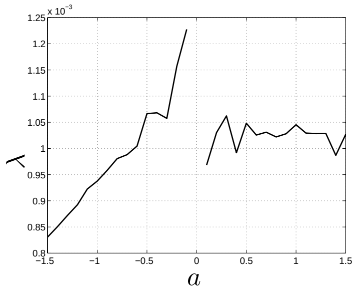

Figure 3: *Graph of different eigenvalues* $\lambda = \frac{\|g(a\Delta x_{\lambda_{max}})\|_{\Delta x_{max}}}{a}$ *obtained by finite difference along dominant vector, that is taking* $\Delta x = a\Delta x_{\lambda_{max}}$.

The value of the dominant eigenvector is $\Delta x_{\lambda_{max}} = [2.5\,10^{-3};\ -2.9\,10^{-3};\ -4.4\,10^{-4};\ 1.4\,10^{-5};\ 6.9;\ -8;\ -1.2]10^{-3}$. The major components are the ones of velocity. This shows that there are only velocity errors after a double support phase and a single support phase. This is due to the fact that the velocity $\dot{\alpha}$ is not controlled during single support. Although the dynamics of α are the slowest to convergence they are relatively fast, since dominant eigenvalue is very small. This is allowed by the over actuated double support phase.

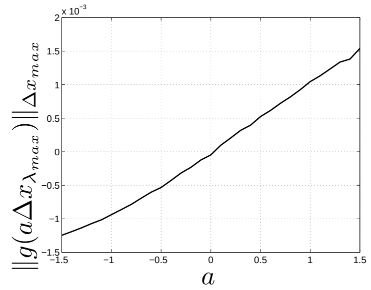

Figure 4: *Poincaré map along the dominant vector around the fixed point, that is taking $\Delta x = a\Delta x_{\lambda_{max}}$.*

7 Conclusion

We described in this paper a numerical strategy to study the orbital stability of a walking gait for a biped without feet. The walking gait is composed of single and double support phases and impacts. The control in double support phase accelerates the convergence of the reference trajectory to the fixed point in the Poincaré map. This point corresponds to the periodic motion. All the process of control is done satisfying the constraints. Numerical simulations show the feasibility of the method. Our perspective is to use the control strategy experimentally on the prototype RABBIT which is the object of our study.

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