Optimization process to design walking cyclic gaits with single and double supports for an underactuated biped

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Abstract This paper deals with a methodology, to design optimal reference trajectories for walking gaits of a five link biped, the prototype *Rabbit*. It has point feet and four actuators in each knee and haunch. *Rabbit* is underactuated in single support since it has no actuated feet and is overactuated in double support. To take into account this characteristic of under-actuation, the reference trajectories for the four actuated joints are prescribed as polynomial in function of the absolute orientation of the stance ankle. There is no impact. The chosen criterion is the integral of the norm 2 of the torques. Different technological and physical constraints are taken into account to obtain a walking such as, the limits of torques, the strictly monotone evolution in time of the absolute orientation of the stance ankle, the existence conditions of solutions of the inverted geometrical model in double support, the unilateral constraints with the ground in the stance leg tips. The optimal process are solved, considering an order of treatment of constraints, according to their importance on the feasibility of the walking gait. Numerical simulations of walking gaits are presented to illustrate this methodology.

1 Introduction

For more than thirty years walking robots and particularly the bipeds have been the object of researches. For example Vukobratovic and co-author in [1] have proposed in 1968 his famous Zero-Moment Point (*ZMP*), for the analysis of a biped gait with feet. In 1977, optimal trajectories [2] are designed for a bipedal locomotion using a parametric optimization. Formal'sky in [3] completely characterized the locomotion of anthropomorphic mechanisms in 1982. Sutherland and Raibert in the paper [4] proposed their principle about virtual legs for walking robots in 1983. Currently Humanoïds such as *Honda* biped in [5] and *HRP2* biped in [6] (Humanoïd Robotics Project 2), which are probably, on the technological point-of-view, the most advanced biped robots, lead to many popular demonstrations of locomotion and interaction with their environnement. In parallel, some researchers, for legged robots with less degrees of freedom, work with the control, the model, the reference trajectories to design walking bipedal gaits more fluid ounce, see for examples [7] where a biped with telescopic legs is studied, [8] where the famous dog *Aibo* from *Sony* is used to design biped gaits, [9] where an intuitive approach is developed for a biped locomotion or [10] where an accurate analysis of the gravity effects is made to give necessary and sufficient conditions to ensure a cyclic walking gait for a biped without feet.

In this paper, the efforts are focused on the design by a parametric optimization of an impactless gait for a planar five-link biped without feet and with four actuators only. This gait is composed of a single support phase and a double-support phase. The originality of the present work is double:

- The four prescribed variables in single support, to overcome the underactuated characteristic of the biped, are function of another generalized coordinate, the absolute orientation at the stance leg ankle. In double support, two actuated joints are prescribed in function of α which is a polynomial function in time.
- There is a classification and a treatment of constraints according to their importance on the feasibility of the walking gait.

The article is organized as follows: the dynamical model of the biped under interest is presented in Section 2 for the single-support phase and double-support phase. Section 3 is devoted to the definition of the reference trajectories, their constraints and their parameters. The design of the optimal gaits with the calculation of the criterion in torque in single support and double support is detailed in Section 4. Some simulation results are shown Section 5. Section 6 contains our conclusion and perspectives.

2 Dynamic model

A planar five-link biped is considered and is composed by a torso and two identical legs with knee and point feet (see a diagram of the studied biped Figure 1).

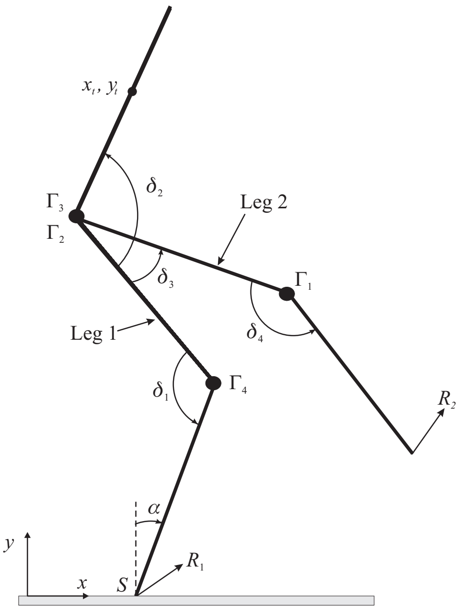

Fig. 1. Biped in the sagittal plane.

There are four identical motors, which drive the haunches and the knees. We note $\Gamma = [\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4]^T$ the torque vector, $q = [\alpha, \delta^T]^T = [\alpha, \delta_1, \delta_2, \delta_3, \delta_4]^T$ the vector composed of the orientation of the stance leg and the actuated joint variables, and $X = [q^T, x_t, y_t]^T$ the vector of generalized coordinates. Components (x_t, y_t) fixe the position of the center of gravity of the trunk.

2.1 General model

The dynamic model is determined from Lagrange's equations and is given by

$$A_e(q)\ddot{X} + H_e(q,\dot{q}) = D_{e\Gamma}\Gamma + D_{e1}(q)R_1 + D_{e2}(q)R_2. \quad (1)$$

The inertia matrix $A_e(7 \times 7)$ of the biped is symmetric and positive definite. The centrifugal, Coriolis and gravity effects are represented by vector $H_e(7 \times 1)$. The torque vector matrix Γ is taken into account by the fixed matrix $D_{e\Gamma}(7 \times 4)$, consisting of zeros and units. $D_{ej}(q)$ is the 7×2-Jacobian matrix converting the ground reactions in the leg tip j into the corresponding joint torques, $(j = 1, 2)$. If the point foot j is in the air, then $R_j = \begin{pmatrix} 0 \; 0 \end{pmatrix}^T$. To take into account of Coulomb dry and viscous frictions, Γ can be written

$$\Gamma = \Gamma - \Gamma_s sign(D_{e\Gamma}^T \dot{q}) - F_v D_{e\Gamma}^T \dot{q} \quad (2)$$

where $\Gamma_s(4 \times 4)$ and $F_v(4 \times 4)$ are diagonal matrices. If the point foot j is in contact with the ground, the position variables X, the velocity variables

$\dot{X}$, and the acceleration variables $\ddot{X}$ are constrained. In order to write these relations, we define the position, velocity and acceleration of the point foot j in an absolute frame. The position of the point foot j is noted $d_j(X)$. By differentiation of $d_j(X)$ we obtain the relation between the velocity $V_j = (V_{jx}\ V_{jy})^T$ of the point foot j and $\dot{X}$,

$$V_j = D_{ej}(q)^T \dot{X} \quad (j = 1, 2) \tag{3}$$

By another differentiation we obtain the relation between the acceleration $\dot{V}_j = (\dot{V}_{jx}\ \dot{V}_{jy})^T$ of the point foot j and $\ddot{X}$,

$$\dot{V}_j = D_{ej}(q)^T \ddot{X} + C_{ej}(q, \dot{q}) \quad (j = 1, 2) \tag{4}$$

Then the contact constraints for the point foot j with the ground are given by the three vector-matrix equations:

$$\begin{cases} d_j(X) = const \\ \quad V_j = 0 \qquad (j = 1, 2) \\ \quad \dot{V}_j = 0 \end{cases} \tag{5}$$

These vector-matrix equations (5) mean that the position of the point foot j remains constant, and then the velocity and acceleration of the point foot j are null. During the double-support phase, both legs are in contact with the ground. Then the dynamic model is formed of both vector-matrix equations, (1) and (5) for $j = 1, 2$.

2.2 A reduced model

Let us assume that the contact between the leg tip 1 and the ground is acting as a pivot: there is no take off and no slipping of this leg tip 1. Then the biped configuration can be described with vector q only. Using Lagrange's equations a new dynamic model is deduced

$$A(\delta)\ddot{q} + H(q, \dot{q}) + G(q) = D_\Gamma \Gamma + D_2(q) R_2 \tag{6}$$

where $A(\delta)(5 \times 5)$ is the symmetric positive inertia matrix of the biped. As the kinetic energy of the biped is invariant under a rotation of the world frame [11], and viewed that ψ defines the orientation of the biped, the 5×5-symmetric positive inertia matrix is independent of this variable, *i.e.* $A = A(\delta)$. Vector $H(q, \dot{q})(5 \times 1)$ represents the centrifugal, Coriolis effects, and $G(q)(5 \times 1)$ is the gravity effects vector. $D_\Gamma(5 \times 4)$ is a constant matrix composed of 1 and 0. $D_2(q)$ is the 5×2-Jacobian matrix converting the ground reaction in the leg tip 2 into the corresponding joint torques. In single support phase on the leg 1, the ground reaction for foot 2 in air is $R_2 = \begin{pmatrix} 0 & 0 \end{pmatrix}^T$. Model (6) allows us to compute easier the control law. However, it is not possible to take into account a single support on the leg 2 with (6). Furthermore we cannot calculate the ground reaction with model (6) only.

2.3 Passive impact model

During the bipedal gait, the impact occurs at the end of a single support phase, when the swing leg tip touches the ground at a time $t = T$. We assume that this impact is instantaneous, passive, absolutely inelastic. Given these conditions, the ground reactions can be considered impulsive forces and defined by Dirac delta-functions $R_j = I_{R_j} \Delta(t - T)$ $(j = 1, 2)$. Here, $I_{R_j} = \left(I_{R_{jx}} \; I_{R_{jy}} \right)^T$, is the vector of the magnitudes of the impulsive reaction in the leg tip j, see [3]. Impact equations can be obtained through integration of the matrix motion equation (1) for the infinitesimal time from $T - 0$ to $T + 0$ at each instantaneous impact. The torques supplied by the actuators at the joints, Coriolis and gravity forces have a finite value, thus they do not influence an impact. Consequently the impact equations can be written in the following matrix form:

$$A_e(q) \left(\dot{X}^+ - \dot{X}^- \right) = D_{e1}(q) I_{R_1} + D_{e2}(q) I_{R_2} \tag{7}$$

The notation + (resp. −) means just after (resp. before) impact.

After an impact several behaviors of the biped are possible. We consider here that both feet remain fixed on the ground, since we are interested in a gait for which a double-support phase is obtained after impact. In this case the passive impact equation (7) must be completed by the two following vector-matrix equations.

$$V_j^+ = D_{ej}(q)^T \dot{X}^+ = 0 \; (j = 1, 2) \tag{8}$$

The passive impact model composed of (7) and (8) allows to compute the seven components of the velocity vector $\dot{X}^+$ and the two impulsive components of each ground reactions I_{R_j}, $(j = 1, 2)$ from the five components of vector q and the seven components of the velocity vector just before impact, $\dot{X}^-$.

The constraints, that must be satisfied to obtain after impact that both feet remain fixed on the ground, will be presented in the next Section 3.

In conclusion, the structure of the dynamical model of the biped changes in function of the different phases of the gait.

3 Definition of the walk and its constraints

Our objective is to design a cyclic bipedal gait. We begin with the presentation of the reference motions in single-support phase and in double-support phase. An impactless bipedal gait is considered because, in [12] numerical results proved that the insertion of an impact with this walking gait for the studied biped is a very difficult challenge. After the choice of parameters, the constraints will be detailed. In the following indices “ss” and “ds” respectively indicate the single-support phase and the double-support phase.

3.1 Reference motion in single support

During the single support, the biped has five degrees of freedom. With the four actuators for the biped, only four output variables can be prescribed. Then the biped is underactuated in single support. In previous experiments, see for example, [7, 13, 14] the ankle angle α of the stance leg changes absolutely monotonically during the single-support phase. Therefore, it is possible to use the angle variable, α instead of time t as an independent variable during the single-support phase of the bipedal gait. Thus to overcome the underactuated property of the biped in single support the four joint variables δ_j are prescribed as polynomial functions of this ankle angle, $\delta_{j,ss}(\alpha)$ $(j = 1, ..., 4)$. The behavior of α is governed by the dynamic model (6). The complete set of configurations during the motion of the biped is defined by this way and it is not necessary to anticipate a duration for this single-support phase, which is the result of the integration of (6). The order of this polynomial functions (9) is fixed at four to specify initial, final and intermediate configurations, plus initial and final joint velocity variables.

$$\delta_{j,ss}(\alpha) = a_{j0} + a_{j1}\alpha + a_{j2}\alpha^2 + a_{j3}\alpha^3 + a_{j4}\alpha^4 \tag{9}$$

Let us note that it would be possible to prescribe other variables as Cartesian variable. In the goal to avoid the problems of singularity of the inverse geometric model in the single support phase, we prefer to work with angular variables only. However some authors, for example [2, 15] use Cartesian coordinates of the hip for the definition of the bipedal gait. The joint variables are then prescribed. However since the biped is underactuated the evolution of angle α must be such that the biped motion satisfies the dynamic model. Let us introduce

$$\begin{aligned} \dot{q}(\alpha, \dot{\alpha}) &= q^* \dot{\alpha} \\ \ddot{q}(\alpha, \dot{\alpha}, \ddot{\alpha}) &= q^* \ddot{\alpha} + q^{**} \dot{\alpha}^2 \end{aligned} \tag{10}$$

where notation $()^*$ means partial derivative in α. Then we have $q^* = [1\, \delta_1^*\, \delta_2^*\, \delta_3^*\, \delta_4^*]$ and $q^{**} = [0\, \delta_1^{**}\, \delta_2^{**}\, \delta_3^{**}\, \delta_4^{**}]$. With relations (9) and (10) a reduced dynamic model of the biped can be described as (see [14])

$$\begin{cases} \dot{\sigma} = -Mg\,(x_G(\alpha) - x_S) \\ \dot{\alpha} = \dfrac{\sigma}{f(\alpha)} \end{cases} \tag{11}$$

M is the biped mass, g the acceleration of gravity, $x_G(\alpha)$ and x_S are respectively the horizontal component of the positions of the biped's mass center and of the foot of the stance leg. σ is the angular momentum around S. Chevallereau et al. [16] have shown from (11) that

$$\frac{d\sigma^2}{d\alpha} = -2Mg\,(x_G(\alpha) - x_S)\, f(\alpha) \tag{12}$$

If α is strictly monotone, the integration of (12) gives

$$\sigma^2 - \sigma_{iss}^2 = -2Mg \int_{\alpha_{iss}}^{\alpha} (x_G(s) - x_S)\, f(s) ds \tag{13}$$

where σ_{iss} is the angular momentum at the beginning of single support characterized by the initial value α_{iss}. Then the dynamics of the biped are completely defined from (11) in function of $\Phi(\alpha) = \sigma^2 - \sigma_{iss}^2$ such as

$$\dot{\alpha} = -\frac{\sqrt{\Phi(\alpha) + f(\alpha_{iSS})^2 \dot{\alpha}_{iSS}^2}}{f(\alpha)} \tag{14}$$

$$\ddot{\alpha} = -\frac{Mgx_G(\alpha) + \frac{df(\alpha)}{d\alpha}\dot{\alpha}^2}{f(\alpha)} \tag{15}$$

From the solution of the differential equation in α (12) and using relations (14) and (15) the numerical simulation to find the optimal motion and the calculation of constraints will be easier. Those relations will also allow us to write the conditions of existence of a motion in α, see (19).

3.2 Reference motion in double support

In double support, the biped has three degrees of freedom. With its four actuators, the biped is overactuated. Then the motion of the biped is completely defined, with three prescribed degrees of freedom. For a question of convenience for the use of the inverse geometric model, the ankle angle, α and both joint variables, δ_j, $(j = 1, 2)$ are prescribed. A polynomial function in time of third-order (16) is chosen to define α. In a concern to be homogeneous with the single support phase we define both joint angular variables δ_j, as polynomial functions of third-order in α. Then initial and final configurations, and initial and final velocities can be defined for these three prescribed variables. The duration of the double support phase is determined a priori.

$$\begin{cases} \alpha(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \\ \\ \delta_j(\alpha) = a_{j0} + a_{j1}\alpha + a_{j2}\alpha^2 + a_{j3}\alpha^3 \end{cases} \tag{16}$$

3.3 Optimization Parameters

A boundary value problem has to be solved to design this cyclic bipedal gait with successive single and double support phases. This boundary value problem depends on parameters to prescribe the initial and final conditions for each phase. Tacking into account the conditions of continuity between the phases and the conditions of cyclic motion we will enumerate now in details on a half step k the minimal number of parameters which are necessary to solve this boundary value problem.

1. Seven parameters are needed to define the initial and final configurations in double support. Then the parameters α_{ids}, $\delta_{1,ids}$, θ_{ids}, α_{fds}, $\delta_{1,fds}$, θ_{fds} and d, the distance between both tips of the stance legs in double support are chosen. The use of the absolute orientation of the trunk, θ (see Figure(2) instead $\delta_{2,fds}$ is easier and does not change the problem.
2. Time, T_{ds} of the double support is given as parameter.
3. The initial velocity of the biped in single support is prescribed by only three parameters, $\dot{\alpha}_{iss}$, $\dot{\delta}^*_{1,iss}$, $\dot{\delta}^*_{2,iss}$. The velocities $\dot{\delta}^*_{3,iss}$ and $\dot{\delta}^*_{4,iss}$are deduced taking into account the null velocity of the leg tip which takes off.
4. The final velocity of the biped in single support is prescribed by only three parameters, $\dot{\alpha}_{fss}$, $\dot{\delta}^*_{1,fss}$, $\dot{\delta}^*_{2,fss}$. The velocities $\dot{\delta}^*_{3,fss}$ and $\dot{\delta}^*_{4,fss}$ are deduced taking into account the absence of impact of the swing leg tip on the ground, which is equivalent to a null velocity of this tip.
5. The intermediate configuration in single support is determined with the five following parameters, α_{int}, $\delta_{1,int}$ and $\delta_{2,int}$ and the coordinates, ($x_{p,int}$ and $y_{p,int}$) of the swing leg tip. Angle α_{int} is fixed equal to $\frac{\alpha_{iss}+\alpha_{fss}}{2}$.

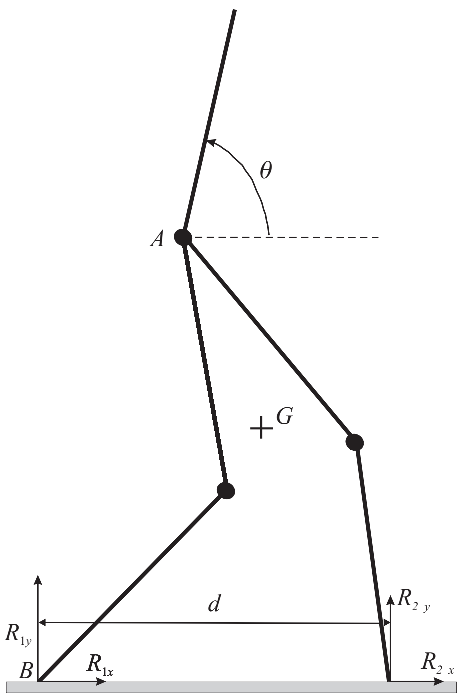

Fig. 2. Biped in the sagittal plane.

Then finally the vector of parameters has seventeen coordinates

$$p = [\alpha_{ids}, \delta_{1,ids}, \theta_{ids}, \alpha_{fds}, \delta_{1,fds}, \theta_{fds}, d, \dot{\alpha}_{iss}, \dot{\delta}^*_{1,iss}, ...$$

$$\dot{\delta}^*_{2,iss}, \dot{\alpha}_{fss}, \dot{\delta}^*_{1,fss}, \dot{\delta}^*_{2,fss}, \delta_{1,int}, \delta_{2,int}, x_{p,int} y_{p,int}]$$

3.4 Constraints

Constraints has to be considered to design nominal gait. We will present them according to their importance on the feasibility of the walking gait.

- Firstly, no motion is possible if distance $d(A, B)$ between the tip of leg 2 and the hip joint, for initial and final configurations of the double support and the intermediate configuration of the single support, is such that

$$d(A, B) > 2 \times l \tag{17}$$

  where l is the common length of the femur and the tibia. In other words, there is no solution with the geometrical model to compute δ_3 and δ_4.
- Constraint (17) is also taken into account during the motion of the biped in double support. The maximum value of $d(A, B)$ in function of α is considered.
- The mechanical stops of joints for initial intermediate and final configurations of each phase and during the motion are

$$\begin{cases} -260^\circ < (\delta_2)_{min} & (\delta_2)_{max} < -110^\circ \\ -260^\circ < (\delta_2 + \delta_3)_{min} & (\delta_2 + \delta_3)_{max} < -110^\circ \\ -230^\circ < (\delta_1)_{min} & (\delta_1)_{max} < -127^\circ \\ -230^\circ < (\delta_4)_{min} & (\delta_4)_{max} < -127^\circ \end{cases}$$

- In double support the monotony condition for variable α is imposed

$$\max_{t \in [0, T_{ds}]} \dot{\alpha}(t) < 0 \tag{18}$$

- In single support, the monotony condition for variable α is imposed by the inequality

$$\Phi_{min} + f(\alpha_{iss})^2 \dot{\alpha}^2_{iss} > 0 \tag{19}$$

  where $\Phi_{min} = \min_{\alpha \in [\alpha_{iss}, \alpha_{fss}]} \Phi(\alpha)$
- In single support it is fundamental to avoid singularity $f(\alpha) = 0$ to simulate one step. Then we define the following constraint

$$\min_{\alpha \in [\alpha_{iss}, \alpha_{fss}]} f(\alpha) > 0 \tag{20}$$

Now the following constraints can be violated to simulate a step. However there are important for experimental objectives.

- Each actuator has physical limits such that

$$\begin{cases} \left(|\Gamma_1^*(\alpha)| - \Gamma_{max}(|\dot{\delta}_1|)\right)_{max} < 0 \\ \left(|\Gamma_2^*(\alpha)| - \Gamma_{max}(|\dot{\delta}_2|)\right)_{max} < 0 \\ \left(|\Gamma_3^*(\alpha)| - \Gamma_{max}(|\dot{\delta}_2 + \dot{\delta}_3|)\right)_{max} < 0 \\ \left(|\Gamma_4^*(\alpha)| - \Gamma_{max}(|\dot{\delta}_4|)\right)_{max} < 0 \end{cases} \tag{21}$$

  The notation $()_{max}$ stands for the maximum value over one step. Function $\Gamma_{max}(*)$ can be deduced from a template, torque actuator/velocity, given by the actuator manufacturer.
- we must take into account of constraints on the ground reaction $R_j = \left(R_{jx}\ R_{jy} \right)^T$ in the tip of the stance leg j, $j = 1$ in single support and $j = 1, 2$ in double support. The ground reaction must be inside a friction cone defined by the friction coefficient f, This is equivalent to write both inequalities

$$\begin{aligned} R_{jx} - fR_{jy} < 0 \\ -R_{jx} - fR_{jy} < 0 \end{aligned}$$

  From these two inequalities, the condition of no take off is deduced:

$$\Rightarrow R_{jy} > 0. \tag{22}$$

- There is also a constraint on the swing leg tip to avoid an impact with the ground during its transfer. This constraint is defined by a parabola function

$$\min_{\alpha \in [\alpha_{iss}, \alpha_{fss}]} \left[y(\alpha) - \left(\frac{x^2(\alpha)}{d^2} - 1 \right) y_{max} \right] > 0$$

  where (x, y) are the coordinates of the swing leg tip.
- Optimal motions are defined for different velocities with the constraint

$$\frac{d}{T_{ss} + T_{ds}} = v \tag{23}$$

  where v is the desired average velocity of the biped. Time T_{ss} of the single support is defined such as the desired average velocity for the locomotion of the biped. The calculation of time T_{ss} of the single-support phase is given by $T_{ss} = \displaystyle\int_{\alpha_{i_{ss}}}^{\alpha_{f_{ss}}} \frac{1}{\dot{\alpha}} d\alpha$

4 Optimal walk

Many values of parameters presented Section 3 can give a periodic bipedal gait satisfying contraints (17)-(23).

Then a parametric optimization process, minimizing a criterion under non-linear constraints, is possible to find a particular nominal motion. Let us define this optimization process

$$\begin{aligned} &\min_{p} C(p) \\ &g_i(p) \leq 0 \quad i = 1, 2, ..., n \end{aligned} \tag{24}$$

where p is the vector of parameters, $C(p)$ is the criterion to minimize with n constraints $g_i(p) \leq 0$ to satisfy. We give now some details about this optimization process and the way to calculate its criterion during the single-support phase and the double-support phase of the nominal researched motion.

4.1 Criterion

To find the nominal motion, criterion C_Γ is considered

$$\begin{aligned} C_\Gamma &= \frac{1}{d} \int_0^{T_{ss}+T_{ds}} \Gamma^T \Gamma dt \\ &= \frac{1}{d} \left(\int_{\alpha_{iss}}^{\alpha_{fss}} \frac{\Gamma^T \Gamma}{\dot{\alpha}} d\mu + \int_0^{T_{ds}} \Gamma^T \Gamma dt \right) \end{aligned} \tag{25}$$

where T_{ss} and T_{ds} are the times of single support and double support. This criterion represents the losses by Joule effects to cover distance d, see [17, 18].

4.2 Single-support phase

From calculation of integral term (13) using the polynomial functions (9), we obtain $\Phi(\alpha) = \sigma^2 - \sigma_{iss}^2$ and velocity $\dot{\alpha}$. Acceleration $\ddot{\alpha}$ can be obtained with relations (14) and (15). Then the dynamics of the underactuated biped in single support is completely defined. The torques are determined from the four last equations of (6)

$$A_{25}(\delta)\ddot{q} + H_{25}(q, \dot{q}) + G_{25}(q) = D_{\Gamma 25}\Gamma \tag{26}$$

Then $A_{25}(4 \times 5)$, $H_{25}(4 \times 5)$ and $D_{\Gamma 25}(4 \times 4)$ are the submatrices of A, H and D_Γ, $G_{25}(4 \times 1)$ is the subvector of G. The invertible matrix $D_{\Gamma 25}$ allows to determine the torque vector Γ. The ground reaction $R_i = (R_{ix}, R_{iy})$ in the tip of the stance leg i are calculated applying Newton's second law in the center of mass G of the biped

$$\begin{aligned} M\ddot{x}_G &= R_{ix} \\ M\ddot{y}_G &= R_{iy} - Mg \end{aligned}$$

where M is the mass of the biped and (x_G, y_G) are the coordinates of G.

4.3 Double-support phase

From relations (16) first, at each step time $\alpha(t)$, $\dot{\alpha}(t)$ and $\ddot{\alpha}(t)$ are calculated as polynomial functions of time, then $\delta_j(\alpha)$, $\dot{\delta}_j(\alpha)$ and $\ddot{\delta}_j(\alpha)$ $(j = 1, 2)$ are determined. To realize the double support, there are an infinity of solutions for the torques because the biped is overactuated. Only three generalized coordinates, for example $\alpha(t)$, δ_1 and δ_2, are necessary to described the motion completely. Then, we can parameterize the solution of torques in function of a variable. To find this variable let us consider Newton's second law in the center of mass G of the biped in double support and the equation of the angular momentum theorem applied in the leg tip 1 on the ground. This theorem, the time derivative of the angular momentum in a fixed point equals the sum of the external momentum forces in this fixed point, leads to an equation which is equivalent to the first line of model (6). Then these three equations are

$$M\ddot{x}_G = R_{1x} + R_{2x} \tag{27}$$

$$M\ddot{y}_G = R_{1y} + R_{2y} - Mg \tag{28}$$

$$A_1(\delta)\ddot{q} + H_1(q, \dot{q}) + G_1(q) = -dR_{2z} \tag{29}$$

where $A_1(1 \times 5)$, $H_1(1 \times 5)$ and are the first line of A and H, $G_1(4 \times 1)$ is the first element of G. Term d is the distance between the two leg tips on the ground. Component R_{2x} does not appear in equation (29) because the ground is assumed to be horizontal and plane. From equations (28) and (29), for a given acceleration of the biped there is only one solution for R_{1z} and R_{2z}, independently of the torques. The torques have an influence only on R_{1x} and R_{2x}. For this reason, a solution for the torques can be founded in function of R_{1x} or R_{2x} as parameter. Let us choose R_{2x} and define the minimization problem with the associated constraint on component R_{2x}

$$\min_{R_{2x}} \Gamma^{*T} \Gamma^*$$

$$\begin{cases} -fR_{1y} - R_{1x} \leq 0 \\ -fR_{1y} + R_{1x} \leq 0 \\ -fR_{2y} - R_{2x} \leq 0 \\ -fR_{2y} + R_{2x} \leq 0 \end{cases} \tag{30}$$

With the four last lines of the vector-matrix equations (6) and (2) a relation between torques Γ^* and R_{2x} can be written

$$\Gamma^* = J - KR_{2x} \tag{31}$$

with $K = D_{\Gamma 25}^{-1} D_{2x\ 25}$ and

$$J = D_{\Gamma 25}^{-1} \left(A_{25}\ddot{q} + H_{25}(q,\dot{q}) + G_{25}(q) - D_{2z\ 25}R_{2z} \right) + \Gamma_s \mathrm{sign}(D_{\Gamma}^T\dot{q}) + F_v D_{\Gamma}^T\dot{q}.$$

The solution $R_{2x\ opt\Gamma}$ to minimize the norm of the torques without constraint is given when $\Gamma^{*T}\frac{\partial \Gamma^*}{\partial R_{2x}} = 0$. Considering equation (31) $R_{2x\ opt\Gamma}$ is given by

$$R_{2x\ opt\Gamma} = \frac{K^T J}{K^T K} \tag{32}$$

defining a minimum value R_{2xinf} and a maximum value R_{2xmax}, the constraints on R_{2x} can be written under the simple form,

$$R_{2xinf} \leq R_{2x} \leq R_{2xmax} \tag{33}$$

Then a solution for the minimization problem (30) is given by three cases

- if $R_{2x\ inf} \leq R_{2x\ opt\Gamma} \leq R_{2x\ sup}$ then $R_{2x} = R_{2x\ opt\Gamma}$,
- if $R_{2x\ opt\Gamma} \leq R_{2x\ inf}$ then $R_{2x} = R_{2x\ inf}$,
- if $R_{2x\ sup} \leq R_{2x\ opt\Gamma}$ then $R_{2x} = R_{2x\ sup}$.

In the case where there is no solution, *i.e* $R_{2xinf} \geq R_{2xmax}$, we choose R_{2x} to minimize the violation of constraints such as

$$R_{2x} = \frac{R_{2x\ inf} + R_{2x\ sup}}{2}$$

4.4 Optimization algorithm

The algorithm *NPSOL*, see [19] from the package *Matlab* is used to solve this optimization problem with its nonlinear constraints. The sequence of treatment of constraints according to their importance is described Figure 3. From level 0 to level 4, the constraints must be satisfied to simulate one step. Others constraints as the maximum velocity of the biped, the limits torques are considered in level 5.

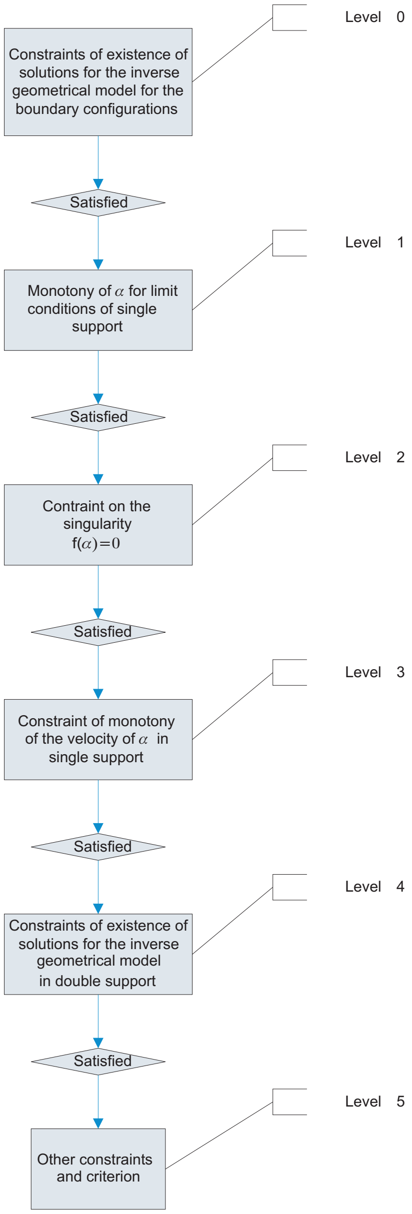

Fig. 3. Sequence of constraints to satisfy before that the step can be defined

Sometimes, while solving the problem (24), the optimization process can ask a value of the criterion or the constraints in a point p_0 where they are not defined. Another point p_M, the closest from p_0 is researched by an intermediate optimization process. For example if constraints $g_{0\,i}(p_0) \leq 0$,

$i = 1, 2, ..., m_0$ are not satisfied, p_M is determined as the solution of the problem

$$\min_p \|p_0 - p\| \qquad g_{0\,i}(p) \leq 0 \quad i = 1, 2, ..., m_0 \tag{34}$$

To solve this intermediate optimization problem (34) and the general optimization problem (24), the gradient in function of the vector of parameters p of the criterion and constraints is necessary. To obtain an efficient algorithm, these gradients are analytically calculated.

5 Simulation results

Figures 4-7 are devoted to a chosen motion velocity for a biped which equals 0.3 m/s. Figure 4 shows that the needed torques for this trajectory are inside the template, motor torque/velocity, given by the manufacturer.

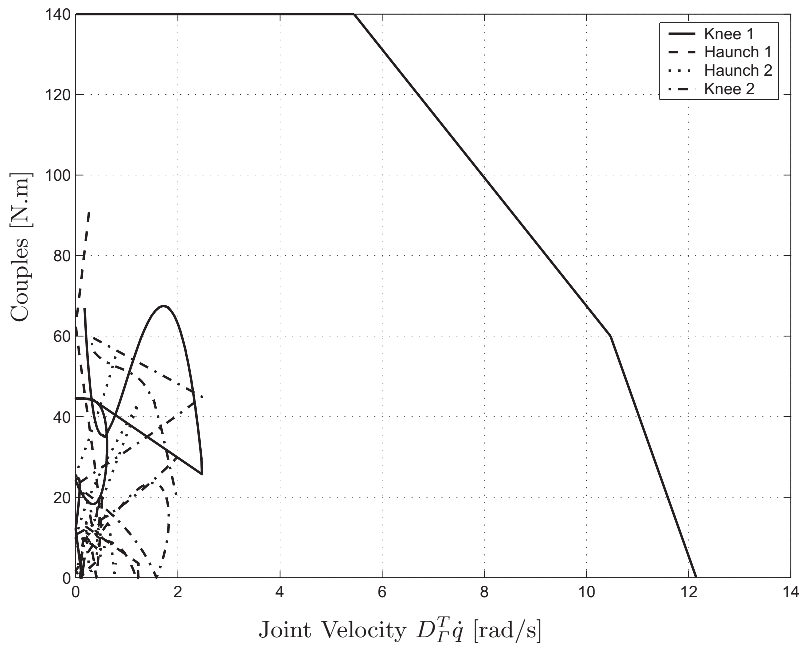

Fig. 4. Velocity versus torque for the actuated joints.

The normal components of the ground reactions in function of time, during one step are presented Figure 5. The constraint of unilateral contact on the leg tip 2 is active because the fixed limit 20 N is reached.

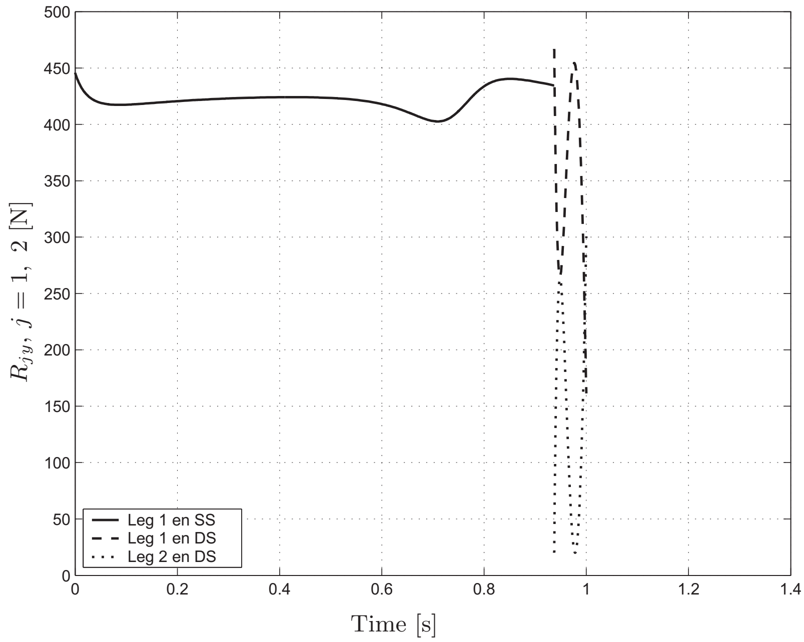

Fig. 5. Normal components in the stance leg tips.

Figure 6 shows in function of time, the evolutions of joint variables δ_1, δ_2, δ_3 and δ_4 in single-support phase and double-support phase.

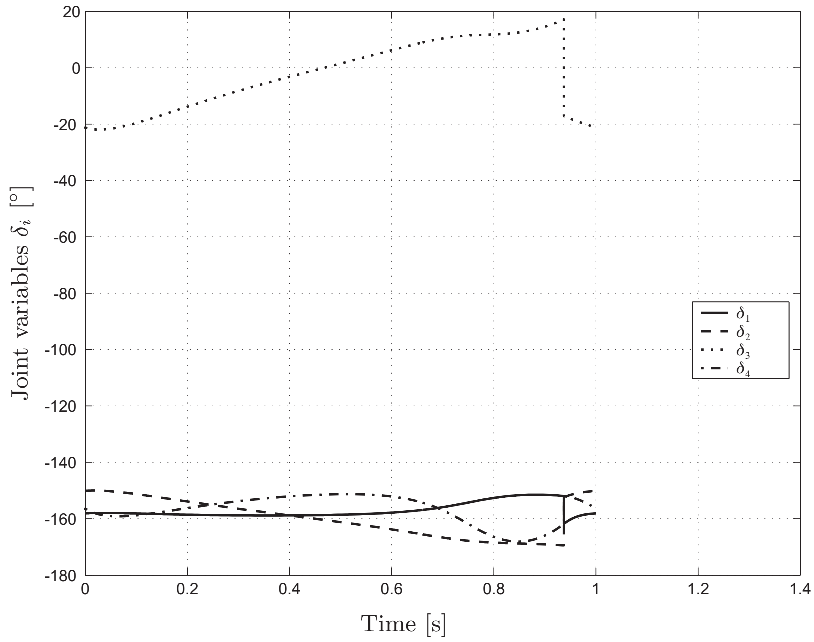

Fig. 6. Evolution of joint variables δ_1, δ_2, δ_3 et δ_4.

Figure 7, the behavior of the variable α is monotone as expected. The discontinuity at the end of the single-support phase is due to the exchange of both legs.

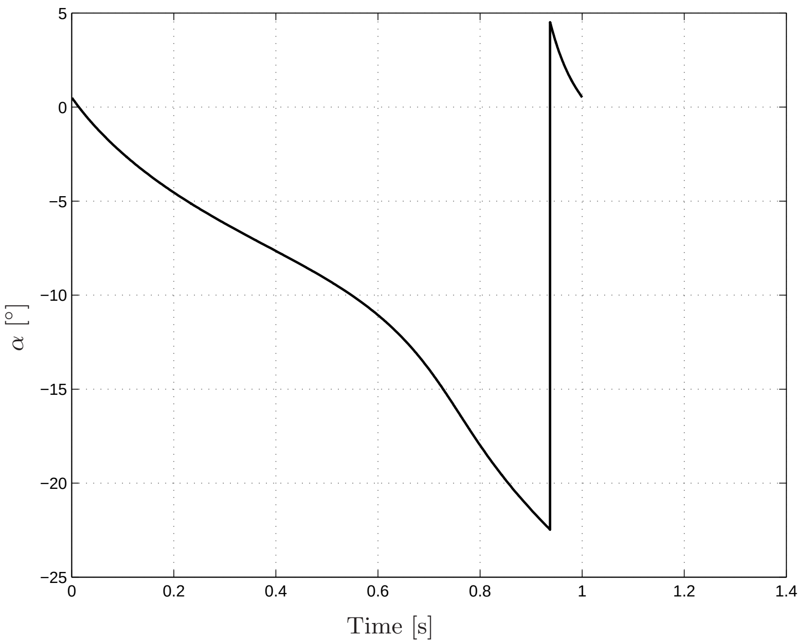

Fig. 7. Evolution of α en function of time.

In conclusion, for this velocity 0.3 m/s of the biped an optimal motion is feasible according to the constraints. Others velocities of motion for the biped are tested with success. Figure 8, different values of criterion C_Γ are presented versus several velocities of motion. A walking biped with single-support phases and double-support phases is more expansive from the point of view of the minimization C_Γ for a velocity greater than 1 m/s. The curves is more smooth if the optimal walk are obtained without to take into account of Coulomb friction. For superior velocities a running gait is more appropriate, (see for example numerical experiments in the paper [17]).

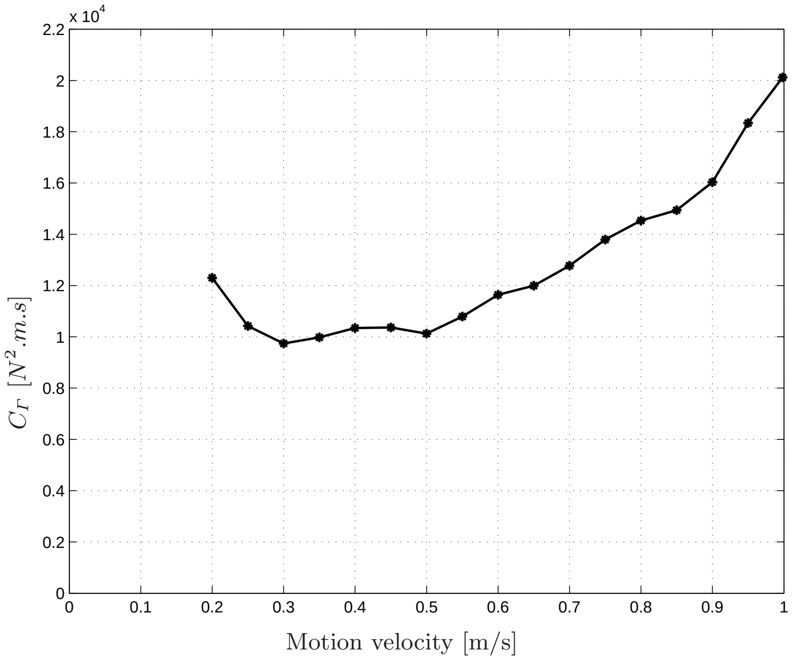

Fig. 8. C_Γ in function of several motion velocities for the biped.

6 Conclusion

An optimization process is proposed to design optimal bipedal gaits for a five-link biped. The walking gaits are composed of single-support phases and double-support phases, but with no impact. The criterion is founded on the integral of the norm 2 of the torques. A sequential procedure is done, taking into account the constraints according their importance to realize a walk step. Coulomb frictions which are nonlinear and discontinuous functions are taken into account because their contribution cannot be neglected. A possible improvement would be to do a piecewise linear approximation of the Coulomb friction, around the discontinuity point. Currently the main drawback of the optimization method we use is that it is not exactly adapted to our problem. Our problem is a semi-infinite problem, that is an optimization problem with constraints that must be satisfied over an interval. We have then adapted our problem by considering the constraints over an interval only at their most constraining point. The optimization problem we then solve is with non-smooth constraints. But we obtained convergence even if *NPSOL* was not designed to cope with such non-smooth problems. To solve our problem, we want in the future to consider an optimization algorithm taking into account or not constraints following they exist or not. We hope also to experiment on prototype

Rabbit these reference trajectories and to extend also this work to a walking biped with more degrees of freedom.

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