Walking with a Double Support Phase

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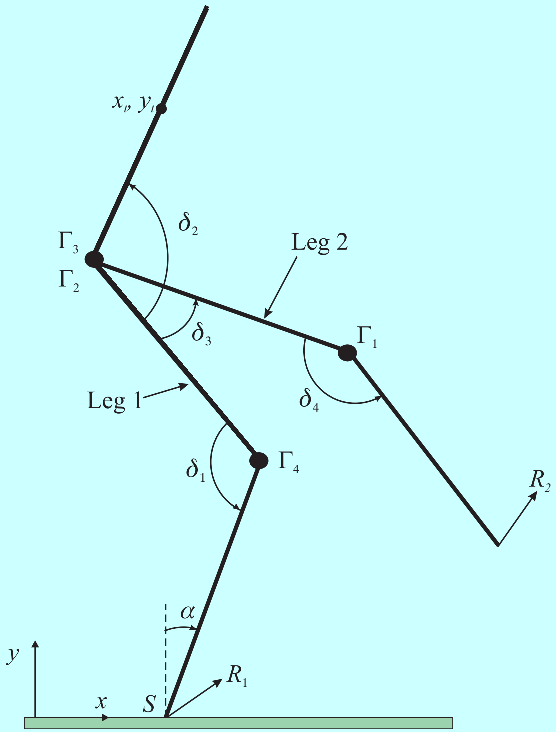
Five link biped's diagram



Seven generalized coordinates
Four actuators.

Under actuated in Single Support:
Five useful generalized coordinates

Over actuated in double support
Three useful generalized coordinates

Dynamic Model

$$A(q)\ddot{X} + H(q,\dot{q}) = D_\Gamma \Gamma + D_1 R_1 + D_2 R_2$$

The tip of the stance legs does not move

$$d_j(X) = const$$
$$V_j = D_j(q)^T \dot{X} = 0 \quad (j = 1,2)$$
$$\dot{V}_j = D_j(q)^T \ddot{X} + H_{cj}(q,q) = 0$$

Definition of a Reference Motion (1/3)

Single support phase

Objectif: To take into account of the under actuation

$$\delta_{i,SS}(\alpha) = a_{i0} + a_{i1}\alpha + a_{i2}\alpha^2 + a_{i3}\alpha^3 + a_{i4}\alpha^4 \quad (i = 1,\ldots,4)$$

These prescribed variables and the undriven variable α lead to a simple semi-inverse system

no term of torques

$$\dot{\sigma} = -Mg(x_G(\alpha) - x_S)$$

$$\dot{\alpha} = \frac{\sigma}{f(\alpha)}$$

COM of the Biped

Definition of a Reference Motion (2/3)

σ Angular momentum $\sigma = \sum_{k=1}^{4} f_k(\delta_i)\dot{\delta}_k + f_5(\delta_i)\dot{\alpha}$

Passive Impact $A(q) = (\dot{X}^+ - \dot{X}^-) = D_1(q) I_{R_1} + D_2(q) I_{R_2}$

If the reference trajectories are exactly tracked a new formulation of the impact:

$$\dot{\alpha}^+ = b\dot{\alpha}^-$$

Definition of a Reference Motion (3/3)

The double support phase

$$\delta_{i,DS}(\alpha) = a_{i0} + a_{i1}\alpha + a_{i2}\alpha^2 + a_{i3}\alpha^3 \ \ (i = 1, 2)$$

The biped is over-actuated: it is possible to use the absolute time

$$\alpha_{DS}(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

Optimization process to obtain a cyclic reference motion with single and double support

Control of α_{DS} in Double Support

Control law

$$\ddot{\alpha} = \begin{cases} \ddot{\alpha}_{max}(\alpha,\dot{\alpha}) & \text{if } \dot{\alpha}(\alpha)-\dot{\alpha}_c(\alpha)<0 \\ \ddot{\alpha}_{min}(\alpha,\dot{\alpha}) & \text{if } \dot{\alpha}(\alpha)-\dot{\alpha}_c(\alpha)>0 \\ \ddot{\alpha}_c(\alpha,\dot{\alpha}) & \text{if } \dot{\alpha}(\alpha)-\dot{\alpha}_c(\alpha)=0 \end{cases}$$

Constraints

Minimal ground reaction

Maximal friction

$$\begin{cases} R_{iy} \geq R_{iy,min} & (i=1,2) \\ -f_{max}R_{iy} \leq R_{ix} \leq f_{max}R_{iy} & (i=1,2) \\ -\Gamma_{max} \leq \Gamma_j \leq \Gamma_{max} & (j=1,\ldots,4) \end{cases}$$

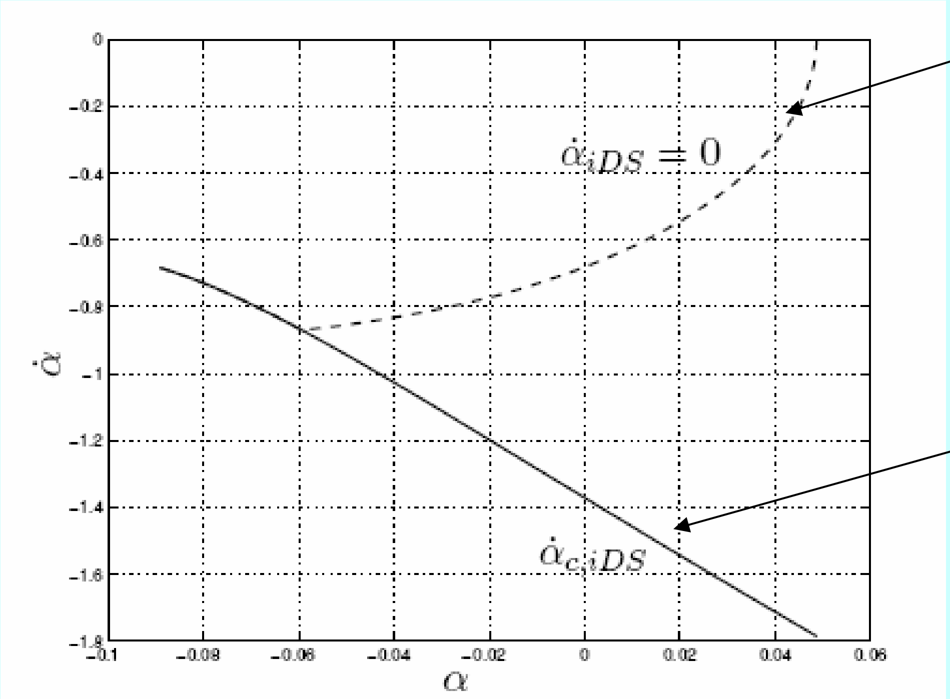
Maximal torque

Two simplex problems

$$\left\{\begin{array}{l} \ddot{\alpha}_{mini}(\alpha,\dot{\alpha}) = \min_{\ddot{\alpha},R_{2x}} \ddot{\alpha} \\ E(\alpha)\ddot{\alpha} + F(\alpha,\dot{\alpha}) + G(\alpha)R_{2x} + P \leq 0 \\ \\ \ddot{\alpha}_{max}(\alpha,\dot{\alpha}) = \max_{\ddot{\alpha},R_{2x}} \ddot{\alpha} \\ E(\alpha)\ddot{\alpha} + F(\alpha,\dot{\alpha}) + G(\alpha)R_{2x} + P \leq 0 \end{array}\right.$$

After several manipulations combining reference motion, dynamic problem and constraints

representative results (1/2)

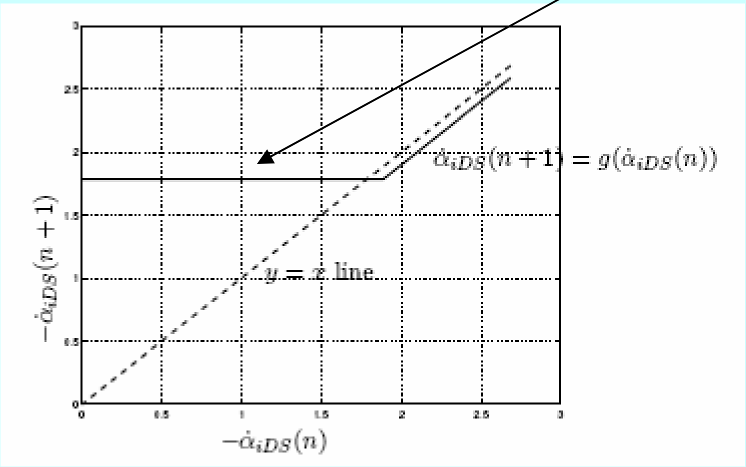


Converging motion starting from null velocity

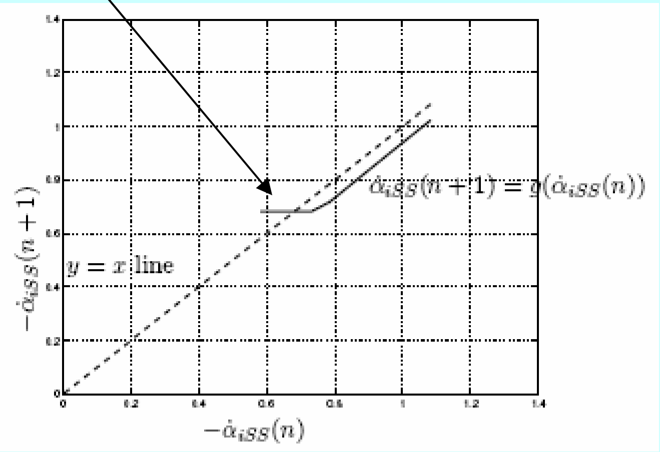
Cyclic motion

representative results (1/2)

Convergence in one step

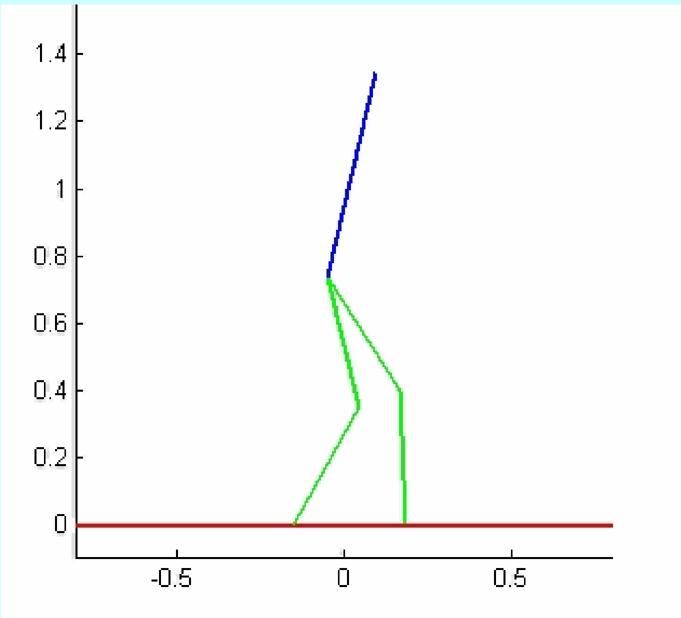


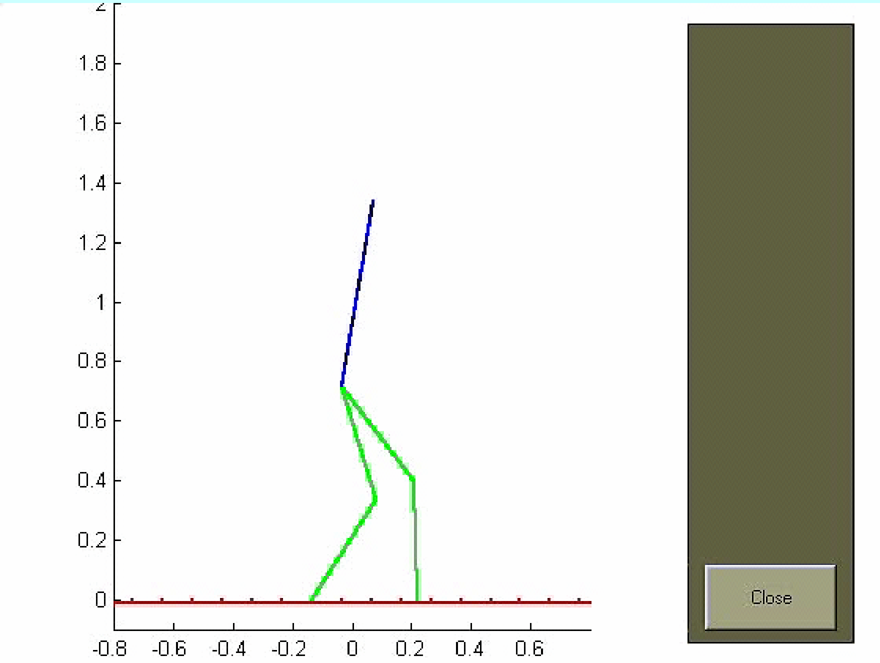
Poincaré map at the beginng of double support phase



Poincaré map at the beginng of single support phase

An Example of Motion





Starting from a stopped position

Conclusion

- An Efficient control law in double support for stabilization of the walk of a biped.
- Determination of the one step convergence.
- It is possible to start from a stooped position.
- Perspective: commutation between different cyclic motions to start, to walk or to stop...