A SIMPLIFIED STABILITY STUDY FOR A BIPED WALK WITH UNDER AND OVER ACTUATED PHASES

S. MIOSSEC, Y. AOUSTIN
Institut de Recherche en Communications et Cybernétique de Nantes, France

ABSTRACT

This paper is devoted to the stability study of a walking gait for a biped. The walking gait is periodic and it is composed of a single support, a passive impact, and a double support. The reference trajectories are described in function of the shin orientation versus the ground of the stance leg. We use the Poincaré map to study the stability of the walking gait of the biped. With the assumption of no perturbation in the tracking of the joint configurations of the biped, the Poincaré map is of dimension one. With a particular control law in double support it is shown theoretically and in simulation that a perturbation error in velocity of the shin angle can be eliminated in one half step only. Therefore, with this possibility, it is shown that it is possible for the biped to reach a periodic regime from a stopped position in one half step.

1 INTRODUCTION

Studies dedicated to bipeds walk can be divided in three categories: static walk, dynamic walk and purely dynamic walk. Static walk consists in walking sufficiently slowly so that dynamics can be neglected. The stability criterion is then a static criterion. Dynamic walk consists in taking partially into account the dynamics of the biped, for example by measuring reaction forces and using the Zero Momentum Point (ZMP) criterion, defined in [1]. For us, this criterion is a necessary but not sufficient criterion for stability of the walk, and is in fact a physical constraint during walk, such as a no take off of the legs constraint. Purely dynamic walk consists in taking into account the dynamics by using a dynamic model of the biped. This approach allows to study theoretically the stability of the walk but is for the moment restricted for simple bipeds, for which the dynamic model is not too complicated. Simple bipeds considered are generally planar bipeds without feet [2-8]. For bipeds without feet the purely dynamic approach is generally necessary since such bipeds are under actuated. For this type of studies, the work can be to generate reference trajectories [5, 6], to study the stability of the walk [2, 3, 4, 7], and to control the biped [2, 8]. Recent theoretical results have been obtained on the stability of a cyclic walk with under actuated single support (SS) phase and with instantaneous double support (DS) phase [4, 7].

What we propose here is a stability study for a walk with under actuated SS phase and non-instantaneous DS phase. The stability study is restricted for a one-dimensional space by using Poincaré map and supposing that the actuated joint reference trajectories are exactly followed. With an appropriate control in DS, it is shown the interest of this phase to improve the stability of the walk. This stability property is due to the over actuation of the DS phase.

We will firstly present the biped, the reference motion and the dynamic model used for the biped. Then we will present Poincaré map, the control law in DS, and some theoretical results for the existence of a zone of convergence to the reference motion in one step. Finally, we will present simulation results.

2 DEFINITION OF A REFERENCE MOTION AND RESULTING SIMPLIFIED MODEL

2.1 Biped presentation

A scheme of the studied biped is presented figure 1 with some notations. It is a biped only moving in the sagittal plan, composed of 5 links (2 tibias, 2 femurs and the trunk), 4 actuated joints (the 2 knees and the hips), and without feet.

We note $\Gamma=[\Gamma_1 \;\; \Gamma_2 \;\; \Gamma_3 \;\; \Gamma_4]^T$ the torque vector, $q=[\alpha \;\; \delta_1 \;\; \delta_2 \;\; \delta_3 \;\; \delta_4]^T$ the joint variables with orientation of the biped in space, and $X=[q^T \; x_t \; z_t]^T$ the configuration, orientation and position vector, where (x_t,z_t) is the position of the center of gravity of the trunk. $R_1=[R_{1x} \;\; R_{1z}]^T$ and $R_2=[R_{2x} \;\; R_{2z}]^T$ are respectively the ground reaction forces of feet 1 and 2.

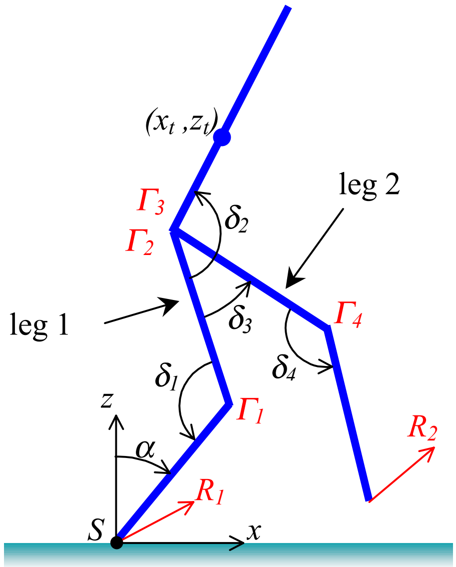

Figure 1 : Scheme of the biped in sagittal plan, and notations

2.2 General considerations

We consider a walk with SS phase, impact, and DS phase.

For this biped the single support phase is under actuated (five degrees of freedom and four actuators). The strategy used is then to prescribed the reference trajectories of the four joint variables δ_j, $(j=1...4)$ in function of the orientation variable of the shin, α. So for a given α, the biped configuration is completely determined in SS. However the behavior of the angular variable α results of the dynamic equations of the under actuated biped.

In DS the biped has three degrees of freedom and is over actuated. It is then possible to prescribe the reference trajectories of only three variables: we choose $\delta_j=\delta_j(\alpha)$, $(j=1,2)$ similarly to the single support phase, and the angular variable α is defined as a polynomial function in time. With these three variables the biped motion is determined completely in DS.

2.3 The Single Support Phase

The reference trajectories of actuated joints, $\delta_{i,SS}$, $(i=1...4)$ are defined as polynomials fourth order in α:

$$\delta_{i,SS}(\alpha) = a_{i0} + a_{i1}\alpha + a_{i2}\alpha^2 + a_{i3}\alpha^3 + a_{i4}\alpha^4 \quad i = 1 \ldots 4 \tag{1}$$

The temporal evolution of α results of the dynamics of the biped.

Supposing $\delta_{i,SS}$ *(i=1…4)* are tracking exactly equations (1) and applying the theorem of the total angular momentum in *S* (the contact point between the stance leg tip of the biped and the ground), and writing expression of angular momentum, we can define a simplified dynamic model (2) to describe the motion of the robot in SS.

$$\begin{cases} \dot{\sigma} = -M.g.x_G(\alpha) \\ \dot{\alpha} = \sigma / f(\alpha) \end{cases} \tag{2}$$

M is the biped mass, *g* the acceleration of gravity, and $x_G(\alpha)$ is the horizontal component of the position of the center of gravity of the biped. σ is the angular momentum around *S*.

The second equation of (2) comes from the expression of the angular momentum given by (3),

$$\sigma = \sum_{k=1}^{4} f_k(\delta_i)\dot{\delta}_k + f_5(\delta_i)\dot{\alpha} \tag{3}$$

where coefficients $f_k(\delta_i)$ *(k=1…5)* depend only of the biped parameters and α, since $\delta_i(\alpha)$ *(i=1…4)* depend only of α, and where $\dot{\delta}_i$ *(i=1…4)* depend only of α and $\dot{\alpha}$ since $\dot{\delta}_i(\alpha,\dot{\alpha}) = \frac{\partial \delta_i(\alpha)}{\partial \alpha}\dot{\alpha}$ *(i=1…4)* by time derivation of (1).

2.4 Simplified Model for the Passive Impact

The impact is considered rigid, passive, instantaneous, with impulsive ground reactions, and with a null restitution coefficient. At the impact instant, an inversion of leg role is performed. The full model of impact is given in [6]. It comes from integration of (7) between just before impact and just after impact. From manipulation of this full impact model with relation (8) and supposing that reference trajectories are exactly tracked, we can obtain the following relation:

$$\dot{\alpha}^+ = b\dot{\alpha}^- \tag{4}$$

The notation "+" means just after impact and "–" just before, *b* is a term depending on the biped configuration at impact and on $\frac{\partial \delta_i}{\partial \alpha}$ *(i=1…4)* just before impact.

2.5 The Double Support Phase

We define the reference trajectories of $\delta_{i,DS}=\delta_{i,DS}(\alpha)$ *(i=1,2)* as polynomials of α:

$$\delta_{i,DS}(\alpha) = a_{i0} + a_{i1}\alpha + a_{i2}\alpha^2 + a_{i3}\alpha^3 \quad i = 1,2 \tag{5}$$

Taking into account that the biped is over actuated α is defined as a polynomial of time:

$$\alpha(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \tag{6}$$

Then the motion of the biped is defined by $\delta_{i,DS}$ *(i=1,2)* and $\alpha(t)$. Since the biped is over actuated, and (5) and (6) are supposed exactly tracked, no dynamic model is needed to obtain the simplified DS motion. But the current study uses the physical constraints during DS (neither slipping nor take off of the feet, and torque limits). We then present the total Lagrangian model during DS, from which constraints are calculated:

$$A(q)\ddot{X} + H(q,\dot{q}) = D_\Gamma \Gamma + D_1(q)R_1 + D_2(q)R_2 \tag{7}$$

Matrix dimensions are $A(7{\times}7)$, $H(7{\times}1)$, $D_\Gamma(7{\times}4)$, $D_i(7{\times}2)$.

The previous model is associated with these position, velocity, and acceleration constraints specifying that the legs remain on the ground:

$$\begin{cases} d_i(X) = cste \\ V_i = D_i(q)^T \dot{X} = 0 \qquad \text{i=1,2} \\ \dot{V}_i = D_i(q)^T \ddot{X} + H_i(q,\dot{q}) = 0 \end{cases} \tag{8}$$

$d_i(X)$ represents the feet position and V_i represents the feet velocity, $i=1,2$.

2.6 Determining the reference motion

We have to find all the coefficients of the polynomial functions (1), (5) and (6). Taking into account that the motion is periodic and is continuous between each phase leads to relations between polynomial coefficients. It is then possible to reduce the number of parameters to 18 (see the principles of this reduction in [5]). An optimization process is used to choose these 18 parameters so that: an energy criterion is minimized, physical constraints are satisfied during the walk (such as no slipping, no take off of the feet, and limit torque). More details for the motion definition are given in [5].

3 POINCARE STABILITY STUDY OF THE REFERENCE MOTION

3.1 Presentation

We only study stability of resulting dynamics after supposing that the actuated joints trajectories are exactly followed (this assumption is a good approximation if we consider a sufficiently efficient control, without too much important perturbations). The resulting dynamics consist in α dynamics given by equations (2) in SS, by $\dot{\alpha}$ discontinuity given by (4) at impact and by α dynamics in DS depending on its control only, presented in the following part.

For this stability study, we use the Poincaré map, which consists in representing the state of a cyclic motion of a system from one period to the following. For the walk considered here, with the previous assumptions, the Poincaré map is a function from only a one-dimension space to a one-dimension space (see also [3, 4, 7]). In this paper we represent the Poincaré map of $\dot{\alpha}$. In the Poincaré map, a cyclic motion is represented by an invariant point, and this cyclic motion is stable if the slope at this point is between –1 and 1. In the next part we will present the control of α in DS so that convergence to the cyclic motion is as fast as possible. Then we will present a proof of existence of a zone of Poincaré map so that convergence is obtained in one step.

3.2 Control of α_{DS} in Double Support

The principle of this control is used in [2] (study in the Poincaré map of the switching condition from transition motion to reference motion) and can be compared to a sliding mode control with a changing gain as large as possible and where the surface is $\dot{\alpha}_c(\alpha)$ (the notation "*c*" designates the reference cyclic motion). The expression of this control is:

$$\ddot{\alpha} = \begin{cases} \ddot{\alpha}_{\max}(\alpha,\dot{\alpha}) & \text{if } \dot{\alpha}(\alpha) - \dot{\alpha}_c(\alpha) < 0 \\ \ddot{\alpha}_{\min}(\alpha,\dot{\alpha}) & \text{if } \dot{\alpha}(\alpha) - \dot{\alpha}_c(\alpha) > 0 \\ \ddot{\alpha}_c(\alpha,\dot{\alpha}) & \text{if } \dot{\alpha}(\alpha) - \dot{\alpha}_c(\alpha) = 0 \end{cases} \tag{9}$$

$\ddot{\alpha}_{\max}(\alpha,\dot{\alpha})$ and $\ddot{\alpha}_{\min}(\alpha,\dot{\alpha})$ are respectively the maximum and minimum possible acceleration to satisfy the physical constraints (no slipping, no take off of the legs, torque limits). We here show the way to determine $\ddot{\alpha}_{\max}(\alpha,\dot{\alpha})$ and $\ddot{\alpha}_{\min}(\alpha,\dot{\alpha})$. The physical constraints considered are the following:

$$\begin{cases} R_{iz} \geq R_{iz,min} & i=1,2 \quad \text{for no take off} \\ -f_{max} R_{iz} \leq R_{ix} \leq f_{max} R_{iz} & i=1,2 \quad \text{for no slipping} \\ -\Gamma_{max} \leq \Gamma_j \leq \Gamma_{max} & j=1..4 \quad \text{for torque limits} \end{cases} \qquad (10)$$

$R_{iz,min}>0$ is the minimal normal ground reaction and f_{max} ($< f$, the real friction coefficient) the maximal friction coefficient of the ground. These constants are security margins.

Our goal is firstly to write these constraints explicitly with respect to α, $\dot{\alpha}$ and $\ddot{\alpha}$, and then to extract the more restrictive constraints on $\ddot{\alpha}$.

Combining (1), (5), (8) into (7) we obtain seven relations between α, $\dot{\alpha}$, $\ddot{\alpha}$, R_{ix}, R_{iz}, Γ_j :

$$A_\alpha(\alpha)\ddot{\alpha} + H_\alpha(\alpha,\dot{\alpha}) = D_\Gamma \Gamma + D_{R_1\alpha}(\alpha)R_1 + D_{R_2\alpha}(\alpha)R_2 \qquad (11)$$

We want to determine the 8 unknown R_{ix}, R_{iz},, Γ_j in function of α, $\dot{\alpha}$ and $\ddot{\alpha}$, whereas there are only seven equations in (11). There are an infinity of solutions that we parameterize by R_{1x} (see [5] for a justification of this choice). So we obtain the following equations:

$$\begin{bmatrix} R_{1z} & R_{2x} & R_{2z} & \Gamma_j \end{bmatrix}^T = B(\alpha)\ddot{\alpha} + C(\alpha,\dot{\alpha},R_{1x}) \qquad (12)$$

With the seven equations (12) we can rewrite the fourteen constraint equations (10) as:

$$C_i(\alpha)\ddot{\alpha} + D_i(\alpha,\dot{\alpha},R_{1x}) \leq 0 \qquad \text{i=1..14} \qquad (13)$$

By taking the more restrictive constraints, the system (13) can be reduced to the two following inequalities : $\ddot{\alpha}_{\min}(\alpha,\dot{\alpha}) < \ddot{\alpha} < \ddot{\alpha}_{\max}(\alpha,\dot{\alpha})$ where

$$\begin{aligned} \ddot{\alpha}_{\max}(\alpha,\dot{\alpha}) &= \min_{i,R_{1x}} \frac{-D_i(\alpha,\dot{\alpha},R_{1x})}{C_i(\alpha)} \quad \text{for } i \text{ so that } C_i(\alpha) > 0 \\ \ddot{\alpha}_{\min}(\alpha,\dot{\alpha}) &= \max_{i,R_{1x}} \frac{-D_i(\alpha,\dot{\alpha},R_{1x})}{C_i(\alpha)} \quad \text{for } i \text{ so that } C_i(\alpha) < 0 \end{aligned} \qquad (14)$$

We now present a representative result of the control in the phase plane $(\alpha,\dot{\alpha})$, see figure 2 (all angles of figures are in radian). For a given initial velocity, less in module than the initial cyclic one, the minimal acceleration is applied and we can see on figure 2 that the $(\alpha,\dot{\alpha})$ motion converges to the cyclic one. Then after the intersection with the cyclic motion, it follows exactly the cyclic motion. We then have a null error at the end of the DS phase.

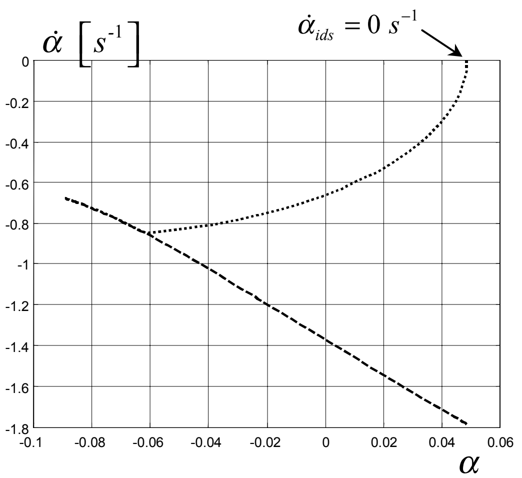

figure 2 : comparison of phase plans, for the cyclic motion and for the motion with a null initial velocity (particular case of start from stop)

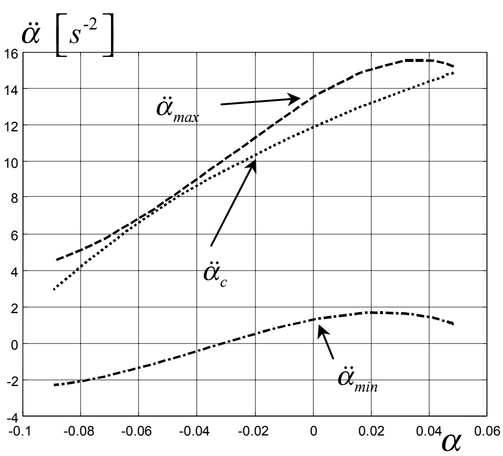

figure 3 : evolution of acceleration $\ddot{\alpha}$, $\ddot{\alpha}_{min}$, $\ddot{\alpha}_{max}$ versus α

Figure 3 shows $\ddot{\alpha}_c$ for the cyclic motion and the corresponding evolution of $\ddot{\alpha}_{\max}(\alpha,\dot{\alpha})$ and $\ddot{\alpha}_{\min}(\alpha,\dot{\alpha})$.

We next present a proof of the existence of an initial velocity zone for which the error at the end of the step is zero.

3.3 Existence of one step convergence zone

To show the existence of a one step convergence zone, we start from the hypothesis that the nominal motion satisfies all strict constraints (13) (see figure 3, the graph $\ddot{\alpha}_c(\alpha)$ between both graphs $\ddot{\alpha}_{\max}(\alpha,\dot{\alpha})$ and $\ddot{\alpha}_{\min}(\alpha,\dot{\alpha})$). The first part of the demonstration is about the convergence of the DS phase. The second part uses results of [7] to extent convergence results on a whole step. An important idea of the demonstration is that we use the phase plan representation $(\alpha,\dot{\alpha})$. Let us note that with the choice of the sign convention for α, its boundaries are such as: $\alpha_{fDS} \le \alpha_{iDS}$ and $\dot{\alpha}_{DS} < 0$. We note "*i*" for initial and "*f*" for final.

Here is the start hypothesis on the DS phase:

Hypothesis: $\forall\ \alpha \in [\alpha_{fDS}, \alpha_{iDS}]$, $\exists\ R_{1x}$, $\forall$ i=1..14 $C_i(\alpha)\ddot{\alpha} + D_i(\alpha,\dot{\alpha},R_{1x}) < 0$

From this hypothesis, by continuity of the constraints (13) with respect to $\dot{\alpha}$ and $\ddot{\alpha}$, we can show that there exist a zone around the cyclic motion for which the strict constraints (13) are still satisfied:

$\exists\ \Delta\dot{\alpha}_{\max} > 0$, $\exists\ \Delta\ddot{\alpha}_{\max} > 0$, $\forall\ \alpha \in [\alpha_{fDS}, \alpha_{iDS}]$, then $\exists\ R_{1x}$, such as $\forall\ \dot{\alpha} \in [\dot{\alpha}_c - \Delta\dot{\alpha}_{\max}, \dot{\alpha}_c + \Delta\dot{\alpha}_{\max}]$, $\forall\ \ddot{\alpha} \in [\ddot{\alpha}_c - \Delta\ddot{\alpha}_{\max}, \ddot{\alpha}_c + \Delta\ddot{\alpha}_{\max}]$, $\forall$ i=1..14, we have

$$C_i(\alpha)\ddot{\alpha} + D_i(\alpha,\dot{\alpha},R_{1x}) < 0 \qquad (15)$$

For the next proof steps we will consider the case $\dot{\alpha}(\alpha_i) > \dot{\alpha}_c(\alpha_i)$. We apply the acceleration, $\ddot{\alpha} = \ddot{\alpha}_c - \Delta\ddot{\alpha}_{\max}$ (results with this acceleration will also be available with control (9) since $\ddot{\alpha}_c - \Delta\ddot{\alpha}_{\max} > \ddot{\alpha}_{\min}(\alpha,\dot{\alpha})$). Then it is possible to show that $\dot{\alpha}(\alpha)$ will cross $\dot{\alpha}_c(\alpha)$. To show this, we study the function:

$$\Delta\dot{\alpha}(\alpha,\dot{\alpha}) = \dot{\alpha}_c(\alpha) - \dot{\alpha} \qquad (16)$$

The derivative of $\Delta\dot{\alpha}(\alpha,\dot{\alpha})$ with respect to α is:

$$\frac{\partial(\Delta\dot{\alpha})}{\partial\alpha} = \frac{d\dot{\alpha}_c}{d\alpha} - \frac{d\dot{\alpha}}{d\alpha} = \frac{d\dot{\alpha}_c}{dt_c}\frac{dt_c}{d\alpha} - \frac{d\dot{\alpha}}{dt}\frac{dt}{d\alpha} = \frac{\ddot{\alpha}_c}{\dot{\alpha}_c} - \frac{\ddot{\alpha}}{\dot{\alpha}} = \frac{\ddot{\alpha}_c}{\dot{\alpha}_c} - \frac{\ddot{\alpha}_c}{\dot{\alpha}} + \frac{\Delta\ddot{\alpha}_{max}}{\dot{\alpha}} \qquad (17)$$

where t_c and t are times depending on whether the cyclic or the current motion are followed.

The value of $\frac{\partial\Delta\dot{\alpha}}{\partial\alpha}(\alpha,\dot{\alpha})$ evaluated on the cyclic motion is:

$$\frac{\partial\Delta\dot{\alpha}}{\partial\alpha}(\alpha_c,\dot{\alpha}) = \frac{\Delta\ddot{\alpha}_{max}}{\dot{\alpha}_c} < 0 \qquad (18)$$

Then by continuity of $\frac{\partial\Delta\dot{\alpha}}{\partial\alpha}(\alpha,\dot{\alpha})$ in $\dot{\alpha}$:

$$\exists\ \varepsilon>0,\ \exists\ \Delta\dot{\alpha}^1_{\max} > 0,\ \forall\ \alpha \in [\alpha_{fDS}, \alpha_{iDS}],\ \forall\ \dot{\alpha} \in [\dot{\alpha}_c, \dot{\alpha}_c + \Delta\dot{\alpha}^1_{\max}],\ \frac{\partial(\Delta\dot{\alpha})}{\partial\alpha} < -\varepsilon \qquad (19)$$

Let us now consider the function $\Delta\dot{\alpha}_2(\alpha,\Delta\dot{\alpha}_{iDS}) = -\varepsilon(\alpha - \alpha_{iDS}) + \Delta\dot{\alpha}_{iDS}$. We have $\frac{\partial\Delta\dot{\alpha}_2}{\partial\alpha} = -\varepsilon$ and $\Delta\dot{\alpha}_2(\alpha_{iDS},\Delta\dot{\alpha}_{iDS}) = \Delta\dot{\alpha}_{iDS}$ and so if we take $\Delta\dot{\alpha}_{iDS} < 0$ sufficiently small, $\exists\ \alpha \in [\alpha_{fDS}, \alpha_{iDS}]$ so that $\Delta\dot{\alpha}_2(\alpha,\Delta\dot{\alpha}_{iDS}) = 0$.

Moreover, by integrating (19) in α we obtain $\forall\ \alpha \in [\alpha_{fDS},\ \alpha_{iDS}]\ \Delta\dot{\alpha} \geq \Delta\dot{\alpha}_2$. Then we have also for a sufficiently small $\Delta\dot{\alpha}_{iDS} < 0$, $\exists\ \alpha \in [\alpha_{fDS},\ \alpha_{iDS}]$ so that $\Delta\dot{\alpha} = 0$.

Finally by generalizing this result whatever the sign of $\Delta\dot{\alpha}_{iDS}$, we can write that:

$$\exists\ \Delta\dot{\alpha}_{iDS,\max} > 0,\ \forall\ \dot{\alpha}_{iDS} \in [\dot{\alpha}_{iDS,c} - \Delta\dot{\alpha}_{iDS,\max}, \dot{\alpha}_{iDS,c} + \Delta\dot{\alpha}_{iDS,\max}],\ \dot{\alpha}_{fDS} - \dot{\alpha}_{fDS,c} = 0 \quad (20)$$

If we now apply results of [7] we have an analytic relation between the velocity at the beginning of the DS $\dot{\alpha}_{iDS}$ and the velocity at the beginning of the previous SS $\dot{\alpha}_{iSS}$:

$$\dot{\alpha}_{iDS} = a(\dot{\alpha}_{iSS}) \quad (21)$$

[7] also provide a condition of the existence of a periodic stable gait for the biped with SS and instantaneous DS. In our case, since DS is over actuated there always exists a periodic stable gate but satisfying the constraints (10) is not then guaranteed.

If we consider the Poincaré map of $\dot{\alpha}_{iDS}$ we will have from (20) and (21) that:

$$\exists\ \Delta\dot{\alpha}_{iDS,\max} > 0,\ \forall\ \dot{\alpha}_{iDS}(n) \in [\dot{\alpha}_{iDS,c} - \Delta\dot{\alpha}_{iDS,\max}, \dot{\alpha}_{iDS,c} + \Delta\dot{\alpha}_{iDS,\max}],$$

$$\dot{\alpha}_{iDS}(n+1) - \dot{\alpha}_{iDS,c} = a(\dot{\alpha}_{iSS}(n+1)) - a(\dot{\alpha}_{iSS,c}) = a(\dot{\alpha}_{fDS}(n)) - a(\dot{\alpha}_{fDS,c}) = 0 \quad (22)$$

If we consider the Poincaré map of $\dot{\alpha}_{iSS}$ we will have from (20) by taking $\Delta\dot{\alpha}_{iSS,\max} = \min\left(\left|a^{-1}\left(\dot{\alpha}_{iDS,c} - \Delta\dot{\alpha}_{iDS,\max}\right) - \dot{\alpha}_{iSS,c}\right|, \left|a^{-1}\left(\dot{\alpha}_{iDS,c} + \Delta\dot{\alpha}_{iDS,\max}\right) - \dot{\alpha}_{iSS,c}\right|\right)$ and from (21) that:

$$\exists\ \Delta\dot{\alpha}_{iSS,\max} > 0,\ \forall\ \dot{\alpha}_{iSS}(n) \in [\dot{\alpha}_{iSS,c} - \Delta\dot{\alpha}_{iSS,\max}, \dot{\alpha}_{iSS,c} + \Delta\dot{\alpha}_{iSS,\max}],$$

$$\dot{\alpha}_{iSS}(n+1) - \dot{\alpha}_{iSS,c} = \dot{\alpha}_{fDS} - \dot{\alpha}_{fDS,c} = 0 \quad (23)$$

We have then demonstrated for both Poincaré representations of $\dot{\alpha}_{iDS}$ and $\dot{\alpha}_{iSS}$ the existence of a null slope zone of the Poincaré map, which corresponds to a zone for which convergence is obtained in one step.

4 SIMULATION RESULTS

We show here the Poincaré map of $\dot{\alpha}_{iss}$ (Figure 4) and of $\dot{\alpha}_{ids}$ (figure 5) with the control law presented. For better representation of Poincaré map we represent opposite of $\dot{\alpha}$, since $\dot{\alpha} < 0$.

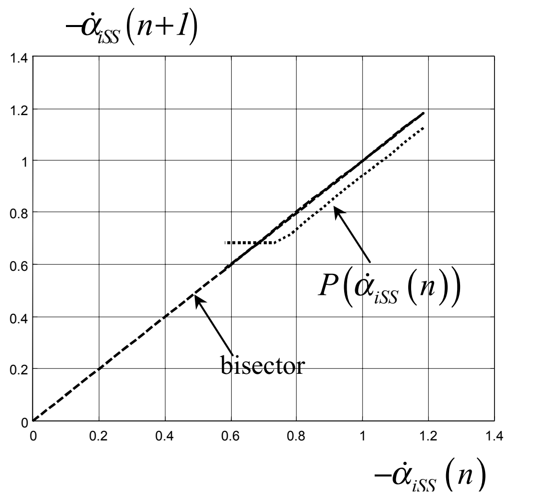

figure 4 : Poincaré map of velocity $\dot{\alpha}$ at the beginning of SS: $\dot{\alpha}_{iSS}(n+1) = P(\dot{\alpha}_{iSS}(n))$

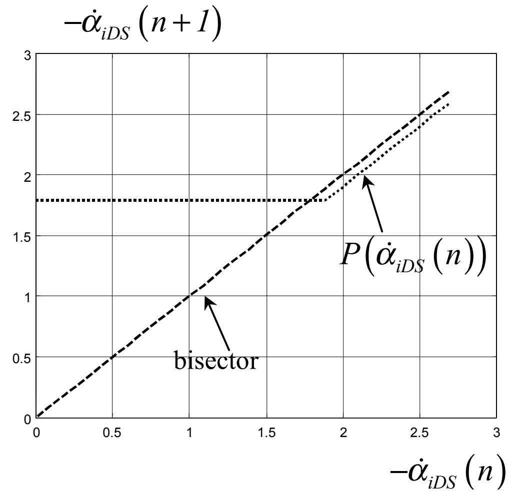

figure 5 : Poincaré map of velocity $\dot{\alpha}$ at the beginning of DS: $\dot{\alpha}_{iDS}(n+1) = P(\dot{\alpha}_{iDS}(n))$

We can see that in both Poincaré maps the null slope zone is large. The presence of such a zone guarantees the convergence from the attraction domain in a finite number of steps. The attraction domain is limited on Poincaré map of $\dot{\alpha}_{iSS}$ for small velocities by the fall back of the biped during SS phase (because of the initial speed too small) and for high velocities by the torque limitation in SS phase.
The attraction domain for Poincaré map of $\dot{\alpha}_{iDS}$ is not limited in small velocities. It means that the biped can start walking from stop. Furthermore, since the Poincaré map slope is zero, this start phase leads directly to the cyclic motion in one step. The limitation in high speed is the same as in the other Poincaré map.
It is also interesting to see that the zone of convergence in one step of the $\dot{\alpha}_{iDS}$ Poincaré map is far larger for small speeds than high speeds. We can see in Figure 3 that it is due to the fact that the distance between the cyclic motion and the constraints is far larger for $\dot{\alpha}-\dot{\alpha}_c < 0$ than for $\dot{\alpha}-\dot{\alpha}_c > 0$. This leads to the idea it would be good to equilibrate distance between cyclic motion and constraints and enlarge it as far as possible.

5 CONCLUSION

We have shown an efficient control law for stabilization of walk of the biped. We thus see the interest of double support phase. We showed the existence of the one step convergence zone. We saw in simulation results that the one step convergence zone is quite large and even allows start from stop. However, a drawback is that the control used is sensible to model errors, since the acceleration constraints depend on the model. To increase robustness, we could increase the security distance with constraints, or use a force sensor measure.
In a future work, we will try to find numerically for a walking motion the largest zone that allows convergence in one step, in order to characterize a good reference motion from a stability point of view. We will also try to stop the biped with the used control law.

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