Walking Gait Composed of Single and Double Supports for a Planar Biped Without Feet

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ABSTRACT

An optimal cyclic reference motion is designed in this paper for an anthropomorphic biped without feet. This gait is composed of a single support, a passive instantaneous impact and a double support. During the single support phase, the under-actuated problem is solved describing the actuated joint variables as polynomial functions of the ankle joint variable, which is non-actuated. During the double support, the biped is an over-actuated system. An energy criterion based on the Joule energy is used for the optimization process.

1. INTRODUCTION

Many papers are devoted to bipeds which are adapted to the human environment, [1]-[13]. One of the main objectives of the biped study is to design cyclic motions, [1]-[4]. To define the cyclic motion, the consumed energy is an important criterion. Therefore, some authors [14] adapt passive motions to obtain feasible motions physically. Other authors, see [1], [3], [8] and [9] obtain optimal motions using this energy criterion. To design the motion, it can be used the cartesian variables [10], the joint variables [11] or the joint torques [8]. The use of the joint variables allows to simplify the computation of the joint torques. The evolution of these variables can be defined with Fourier series [11], with Van der Pol's oscillators [12], or with polynomial functions, of time [3] or of a configuration variable, [6] and [13].
We propose a cyclic gait which is composed of a single support, a passive instantaneous impact and a double support. To solve the under-actuated problem in single support, the four actuated joint variables are specified as polynomial functions of the ankle joint. This ankle joint variable is the non-actuated independent variable, which is experimentally measurable. In double support, the biped is over-actuated. Three joint variables are necessary to define the biped configuration. Therefore we prescribe three independent joint variables. Two variables

are chosen as polynomial functions of the ankle joint variable, which is a polynomial function of time. The introduction of double support phase allows, with the on line modification of the reference motion in velocity, to respect the cyclic conditions, despite velocity errors after the passive impact.
The organization of the paper is the following. Section 2 describes the dynamic model. The section 3 is devoted to the motion definition. The optimization problem is detailed in section 4. The results are presented section 5. The section 6 offers our conclusions and perspectives.

2. BIPED MODELISATION

The biped motion takes place in the sagittal plane. It consists of a torso and two identical legs with knees. All links have mass, are rigid, and are connected with revolute joints. Its contact with the ground is punctual. Four actuators are located in the hips and in the knees.
The configuration variables, the joint torques, and the ground reactions in the leg tips are presented in Figure 1. The torque vector is noted: $\Gamma=[\Gamma_1\ \ \Gamma_2\ \ \Gamma_3\ \ \Gamma_4]^T$. The vector of the joint and orientation angles of the biped is: $q=[\alpha\ \ \delta_1\ \ \delta_2\ \ \delta_3\ \ \delta_4]^T$. With $X=[q^T\ \ x_t\ \ z_t]^T$, where (x_t,z_t) are the coordinates of the trunk mass center, the position and the configuration of the biped are defined in the sagittal plane.

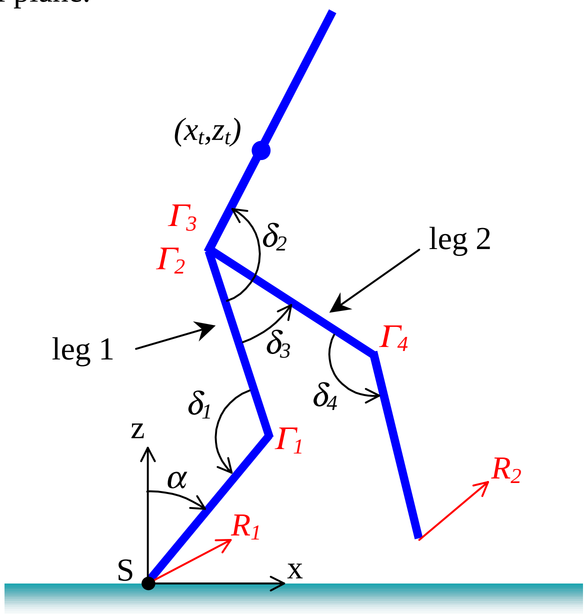

Figure 1 Biped Scheme with notation conventions

We present now the three phases with their models.

2.1. Single Support Phase

There are a stance leg (leg1) and a swing leg (leg2). Therefore the biped dynamic model is:

$$A(q)\ddot{X}+H(q,\dot{q})=D_\Gamma \Gamma+D_1(q)R_1 \qquad (1)$$

$A(7\times7)$ is the inertia matrix; $H(7\times1)$ is the vector of gravity, centrifugal and Coriolis effects; $D_\Gamma(7\times4)$ takes into account the torque effects; $D_1(7\times2)$ is relative to the effect of the ground reactions in the stance leg tip. Equation (2) is the connection condition in acceleration between the ground and the stance leg tip,

$$\dot{V}_1 = D_1(q)^T \ddot{X} + H_1(q,\dot{q}) = 0\,, \tag{2}$$

V_1 is the velocity of the tip of leg 1; $H_1(2\times1)$ is the matrix column $\dot{D}_1(q)^T \dot{X}$. (2) is deduced from the geometric connection condition by two time derivations.
Therefore the fact that the stance leg tip does not take off leads to a constraint on the acceleration vector $\ddot{X}$. Then, there are 5 independent components of $\ddot{X}$, whereas 4 joints are actuated. So, the biped is under-actuated in single support.

2.2. Passive Impact Phase

Passive instantaneous impact with no take off of both leg tips is described by equations (6). This impact is assumed impulsive and inelastic [5]. The ground reactions are described by Dirac functions with intensities I_{R_j}, $j = 1,2$ at time instant T_{ss} of the impact (T_{ss} defines also the time duration of the single support):

$$R_j(T_{ss}) = I_{R_j}\delta(t - T_{ss}) \quad j = 1,2\,. \tag{3}$$

$$\begin{aligned} &A(q)\left(\dot{X}^+ - \dot{X}^-\right) = D_1(q)I_{R_1} + D_2(q)I_{R_2} \\ &V_j^+ = D_j^T(q)\dot{X}^+ = 0 \quad j = 1,2 \end{aligned} \tag{4}$$

The systems (4) is composed of 11 independent equations. $\dot{X}^-$(just before impact) is given. The 11 unknown variables are $\dot{X}^+(7\times1)$ (just after impact) and the impulsive ground reactions $I_{R_j}(2\times1) \quad j = 1,2$.

2.3. Double Support Phase

In double support, both legs 1 and 2 are stance legs. Then the dynamic model of the biped is:

$$A(q)\ddot{X} + H(q,\dot{q}) = D_\Gamma \Gamma + D_1(q)R_1 + D_2(q)R_2 \tag{5}$$

$$D_j(q)^T \ddot{X} + H_j(q,\dot{q}) = 0 \quad j = 1,2 \tag{6}$$

In double support, the biped is over-actuated since the acceleration vector $\ddot{X}$ has 3 independent components (due to constraints (6)) and there are 4 joint torques. Therefore there is an infinity of torque vectors to realize a desired acceleration $\ddot{X}$ compatible with (6).

3. WALKING GAIT DEFINITION

Due to cyclic gait, it is possible to consider one half step only, and to extend the results to the complete step, just by swapping the role of both legs.
The studied half step is composed of the single support on stance leg 1 with swing leg 2, the impulsive impact of the swing leg 2 with the ground and the double support with rear leg 2 and front leg 1 (so leg numbers are inverted at the instant of impact).
Firstly, we present the motion definition and the boundary conditions independently for each phase. Boundary conditions allow to know the entire motion. Then the cyclic conditions and the connections between phases are introduced.

3.1. Single Support Phase

In single support, it is possible to define trajectory for the joint and the orientation variables q only. After, we can deduce from (2), the position, the velocity and the acceleration of the mass center of the trunk.

Taking into account that the single support phase is under-actuated of degree 1, we choose to prescribe the trajectory of the four active joints. Therefore in this case the free variable, the ankle angle α will verify the dynamic behavior of the biped.

The reference trajectory for each joint variable δ_i i=1…4 is specified in function of α. The objective is to avoid to define the absolute single support time and to obtain the desired final configuration with a control in closed loop. With this definition of α and δ_i i=1…4, the motion of the biped is totally specified.

We choose a polynomial form for δ_i i=1…4. Then we have to define the coefficients of the polynomial functions with the following boundary conditions (where notation ss refers to single support),

1. initial configuration α_{iss} , $\delta_{i,iss}(\alpha_{iss})$ i=1…4, and velocities $\dot{\alpha}_{iss}$, $\dot{\delta}_{i,iss}(\dot{\alpha}_{iss})$ i=1…4,
2. final configuration α_{fss} , $\delta_{i,fss}(\alpha_{fss})$ i=1…4, and velocities $\dot{\alpha}_{fss}$, $\dot{\delta}_{i,fss}(\dot{\alpha}_{fss})$ i=1…4,
3. intermediate configuration of the biped α_{int}, $\delta_{i,int}(\alpha_{int})$ i=1…4.

Let us note that these boundary values will be useful to express easier in part 3.3 the cyclic condition of the gait and the connection between the different phases. Then there are five conditions for each actuated variable and in this case all δ_i i=1…4 are 4-degree polynomials:

$$\delta_{i,ss}(\alpha) = a_{i0} + a_{i1}\alpha + a_{i2}\alpha^2 + a_{i3}\alpha^3 + a_{i4}\alpha^4 \quad i = 1...4 \tag{7}$$

With this definition of the single support we can obtain for the biped the angular momentum theorem equation with respect to the contact point S between the stance leg and the ground, see Figure 1, [13]:

$$\begin{cases} \dot{\sigma} = -Mgx_G(\alpha) \\ \dot{\alpha} = \sigma / f(\alpha) \end{cases} \tag{8}$$

with M is the mass of the biped; g is the gravity acceleration; $x_G(\alpha)$ is the abscissa of mass center biped G in absolute frame $(S,\vec{x},\vec{z})$; σ is the angular momentum of the biped in point S.

Function $f(\alpha)$ which depends on the biped configuration is homogenous to an inertia momentum.

The evolution of ankle angle α is obtained from the integration of the system (8).

A parallel may be do with the definition of the zero dynamics: The dynamic of α is the internal dynamic of the system when the tracking error in δ_i i=1…4 are zero (see [15]).

3.2. Double Support Phase

During double support phase we choose to prescribe the trajectory of three independent variables α, δ_1 and δ_2. After, it is possible to deduce the trajectory of joint variables δ_3, δ_4 and trunk center of gravity coordinates (x_t,z_t) from the system (6).

To adopt an homogenous strategy with the one of single support, the trajectories of δ_1 and δ_2 are specified in function of α. However, due to the fact that the biped is over-actuated we can choose α as a polynomial function of time.

The coefficients of the different polynomial functions are deduced from the following boundary conditions (where notation ds refers to double support):

1. initial configuration α_{ids} , $\delta_{i,ids}(\alpha_{ids})$ i=1,2, and velocities $\dot{\alpha}_{ids}$, $\dot{\delta}_{i,ids}(\dot{\alpha}_{ids})$ i=1,2,
2. final configuration α_{fds} , $\delta_{i,fds}(\alpha_{fds})$ i=1,2, and velocities $\dot{\alpha}_{fds}$, $\dot{\delta}_{i,fds}(\dot{\alpha}_{fds})$ i=1,2.

Four conditions per variable allow to defined a polynomial function of degree 3:

$$\delta_{i,ds}(\alpha) = a_{i0} + a_{i1}\alpha + a_{i2}\alpha^2 + a_{i3}\alpha^3 \text{ i=1,2} \quad (9)$$

We use a time polynomial function of degree 2 for α:

$$\alpha_{ds}(t) = a_0 + a_1 t + a_2 t^2 \quad (10)$$

The time of the double support is computed from the boundary conditions on α.

3.3. Connections of the Phases and Cyclic Conditions

The cyclic conditions are connection conditions between the end of a half step and the beginning of the same half step. Connection conditions consist in the continuity of the positions and the velocities between each phase of the walking gait. So these conditions are relations between boundary conditions of single support and double support. Taking into account all of these conditions simplify the optimization process (see part 4.3).

4. BIPED MOTION BASED ON AN OPTIMIZATION PROCESS

In this section we detail the optimization process.

4.1. The Constraints

Three kinds of constraints can be considered. Firstly there are the own technologic limits of the biped: the joint stops, the maximum torque values of the actuators. There are the physical constraints during the biped motion: the no taking off and the no sliding of the stance leg tips during the single support, the instantaneous impact and the double support. A last condition is added by choice: the tip of the swing leg in single support must be over a parabolic trajectory to avoid an undesired impact before the end of this phase.

4.2. The Criterion

The chosen criterion describes the dissipated energy by Joule effects in the actuators, [3],

$$C = \frac{1}{d} \int_0^{T_{hs}} \|\Gamma\|_2^2 \, dt \quad (11)$$

where d and T_{hs} are the length and the time of a half step.

The torques here are obtained for each state vector of the biped during its motion by inversion of the system (1) in single support and of the system (5) in double support. In single support, the solution is unique. In double support, there is one degree of over-actuation (see part 2.3). Therefore, at each instant there is an infinity of solutions for the torque vector. We compute the solution that minimizes the norm $\|\Gamma\|_2^2$ and verify the no sliding constraints.

4.3. The Parameters of the Optimization Process

The parameters of the optimization process are defined so that all boundary conditions can be obtained from them with the equations of connection conditions between phases, of cyclic conditions and of contact of leg tips with ground. An important purpose for the optimization efficiency should be to choose the parameters so that the criterion will be the closest to a convex function of the parameters. Since it is not obvious, Our goal has been to obtain the least possible number of parameters, only. We have chosen the following parameters:

1. The final configuration in double support, defined by 4 parameters, α_{fds}, $\delta_{1,fds}$, $\delta_{2,fds}$ and d. d is distance between both stance leg tips, which is constant in double support.
2. The final velocities of the double support defined by 3 parameters $\dot{\alpha}_{fds}$, $\dot{\delta}_{1,fds}$ and $\dot{\delta}_{2,fds}$.
3. The intermediate configuration in single support, defined by 5 parameters, α_{int}, $\delta_{1,int}$, $\delta_{2,int}$ and the coordinates of the swing leg tip $(x_{p,int}, z_{p,int})$. $z_{p,int}$ is fixed to 5 cm.
4. The initial configuration of the double support, defined by α_{ids}, $\delta_{1,ids}$, $\delta_{2,ids}$ and d.
5. The final velocities of the single support, defined with the five variable values $\dot{\alpha}_{fss}$, $p_i = \dot{\delta}_{i,fss} / \dot{\alpha}_{fss}$ i=1…4. The usefulness of values p_i is obvious if we consider the time derivative of (7),

$$\dot{\delta}_{i,fss} = \left(a_{i1} + 2a_{i2}\alpha_{fss} + 3a_{i3}\alpha_{fss}^{\ 2} + 4a_{i4}\alpha_{fss}^{\ 3}\right)\dot{\alpha}_{fss} \qquad i = 1...4 \qquad (12)$$

Therefore, we determine the polynomial coefficients with p_i i=1…4, transforming (12) in (13), because the velocity $\dot{\alpha}_{fss}$ is computed in line with the integration of the system (8),

$$p_i = a_{i1} + 2a_{i2}\alpha_{fss} + 3a_{i3}\alpha_{fss}^{\ 2} + 4a_{i4}\alpha_{fss}^{\ 3} \qquad i = 1...4 \qquad (13)$$

The entire final biped state just before passive impact is known only after the integration of the single support. The initial conditions of the double support are deduced from the passive impact and then, algebraically from the final conditions of the single support. Then, it is necessary to begin the simulation of the studied half step with the single support.

An important fact is that the final conditions of the double support are independent of the final conditions of the single support. Therefore any modification of the final single support state, except the distance between the leg tips, can be taken into account during the double support by a motion modification, provided constraints are still verified. This could be an interesting property in closed loop.

To satisfy the no sliding constraints and the taking off constraints in the instant of the passive impact we impose a particularity to our motion: the velocity of the swing leg tip is put zero at the instant of impact. In fact it is equivalent to the assumption of no impact [5], [9].

This null velocity condition on the swing leg tip at impact induces expression (14) which describes a condition on the joint variables of the biped,

$$J_{leg2}(\alpha_{fss})\begin{bmatrix}1 & p_1 & p_3 & p_4\end{bmatrix}^T = \begin{bmatrix}V_{1x} & V_{1z}\end{bmatrix}^T = \begin{bmatrix}0 & 0\end{bmatrix}^T \qquad (14)$$

It is interesting to notice that p_2 parameter corresponding to the trunk does not have any effect on the swing leg tip. The two equations of system (14) allow to reduce the number of parameters. We choose among parameters p_i i=1…4, p_1 and p_2.

In conclusion, we have 16 parameters to design the cyclic motion of the biped.

5. PRESENTATION OF RESULTS

An optimal cyclic gait is presented. All the constraints are satisfied. C is 7919 N^2.m.s; the mean speed of the biped is 0.44 m.s^{-1}; the length of the half step d = 0.36 m; the times of the double support, T_{ds} = 73 ms, of the single support, T_{ss} = 742 ms, of the half step T_{hs} = 814 ms.

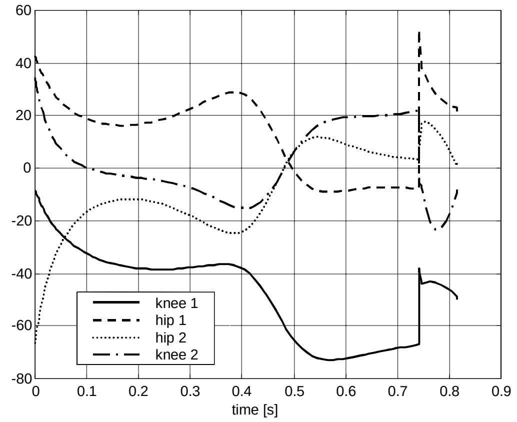

Figure 2 Joint torques [N.m] during one half step

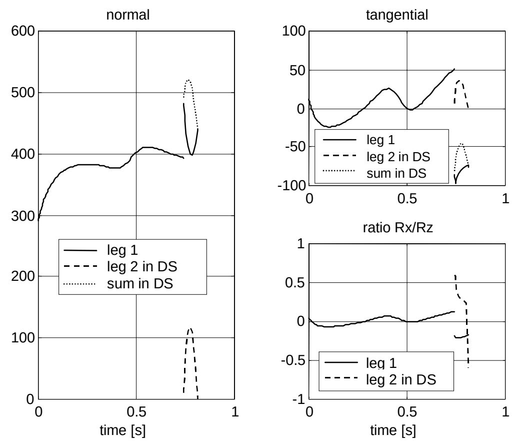

Figure 3 Ground reaction [N] : normal, tangential forces and friction coefficient

On Figure 2 and Figure 3 there are discontinuities between the single support phase and the double support phase. Limit on the maximal value, 150N.m of the torque is satisfied, Figure 2. Figure 3, the vertical components of the ground are strictly positive, in single support and in double support phase. We can see that ratio of the tangential reaction per the normal reaction, which is the friction coefficient, is indeed in limit interval *[-f,f]*, with chosen value f=2/3. The integral equations (15) and (16) represent the mean of acceleration of the center of gravity of the biped in axis x, noted γ_x , and z, noted γ_z , also related with ground reactions:

$$\frac{1}{T_{hs}} \int_0^{T_{hs}} \gamma_z dt = \frac{1}{T_{hs}} \int_0^{T_{hs}} \left(R_{1z} + R_{2z}\right) dt + \frac{1}{T_{hs}} \left(I_{R_{1z}} + I_{R_{2z}}\right) = M.g \quad (15)$$

$$\frac{1}{T_{hs}} \int_0^{T_{hs}} \gamma_x dt = \frac{1}{T_{hs}} \int_0^{T_{hs}} \left(R_{1x} + R_{2x}\right) dt + \frac{1}{T_{hs}} \left(I_{R_{1x}} + I_{R_{2x}}\right) = 0 \quad (16)$$

In our case there is no impulsive impact: $I_{R_{1x}} = I_{R_{2x}} = I_{R_{1z}} = I_{R_{2z}} = 0$. There is a coherence of this calculus with the simulation results. Indeed, the mean value of the sum of the ground reactions obtained by the simulation of one half step is equal to $M.g$=392.4 N for the vertical component and is equal to 0 N for the horizontal component within a very good precision. We can also see on Figure 3 that the ground reactions are effectively around these mean values.

This results, are worse than the ones presented in [3] with the same energy cost criterion. The assumed reasons are the following:

1. The insertion of the double support and the condition of null velocity of the swing leg tip at impact slow down the biped and increase the needed mechanical energy,
2. The use of the angle of the ankle joint to define the variables trajectories of the four active joints in place of time may increase the number of local minimas of the criterion and by this way may reduce the optimization efficiency.

Nevertheless, it is assumed that better stability properties will be obtained, thanks to the possibility of modifying the motion in double support.

6. CONCLUSION

In this paper we designed an optimal walking cyclic gait for a biped. Each half step is composed of a single support, a passive impact and a double support. The active joint is described with the free ankle joint to avoid to specify the single support time and to be sure of the final configuration of the single support. The biped is under-actuated in single support, then the ankle joint will verify the dynamic behavior of the biped. In double support, the biped is over-actuated therefore, we can describe the angle of the ankle as a polynomial function of time. Technologic constraints and unilateral constraints of the biped motion are taken into account. The obtained results look us reasonable.

The perspective of this work is to test this motion in simulation with a closed loop control. The Rabbit prototype from the Project ROBEA (Robotique et Entités Artificielles), named “Commande pour la marche et la course d’un robot bipède” will be used to test all this work.

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